2024

AP

AP® Calculus BC

Sample Student Responses and Scoring Commentary

Inside:

Free-Response Question 1

- ☑ Scoring Guidelines
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Part A (AB or BC): Graphing calculator required

Question 1 — 9 points

General Scoring Notes

The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

t (minutes)	0	3	7	12
$C(t)$ (degrees Celsius)	100	85	69	55

The temperature of coffee in a cup at time t minutes is modeled by a decreasing differentiable function C, where $C(t)$ is measured in degrees Celsius. For $0 \le t \le 12$, selected values of $C(t)$ are given in the table shown.

	Model Solution	Scoring	
(a)	Approximate $C'(5)$ using the average rate of change of C over the interval $3 \le t \le 7$. Show the work that leads to your answer and include units of measure.		
	$C'(5) \approx \frac{C(7) - C(3)}{7 - 3} = \frac{69 - 85}{4} = -4$ degrees Celsius per minute	Estimate with supporting work	**1 point**
		Units	**1 point**

Scoring notes:

- To earn the first point a response must include a difference and a quotient as the supporting work.
- $\frac{-16}{7-3}$, $\frac{69-85}{7-3}$, or $\frac{69-85}{4}$ is sufficient to earn the first point.
- A response that presents only units without a numerical approximation for $C'(5)$ does not earn the second point.
- The second point is also earned for “degrees per minute” attached to a numerical value.

Total for part (a) 2 points

(b) Use a left Riemann sum with the three subintervals indicated by the data in the table to approximate the value of $\int_0^{12} C(t)\,dt$. Interpret the meaning of $\frac{1}{12}\int_0^{12} C(t)\,dt$ in the context of the problem.

$\int_0^{12} C(t)\,dt \approx (3-0)\cdot C(0) + (7-3)\cdot C(3) + (12-7)\cdot C(7)$ $= 3\cdot 100 + 4\cdot 85 + 5\cdot 69 = 985$	Form of left Riemann sum	**1 point**
	Estimate	**1 point**
$\frac{1}{12}\int_0^{12} C(t)\,dt$ is the average temperature of the coffee (in degrees Celsius) over the interval from $t = 0$ to $t = 12$.	Interpretation	**1 point**

Scoring notes:

- Read "$=$" as "$\approx$" for the first point.
- To earn the first point at least five of the six factors in the Riemann sum must be correct. If any of the six factors is incorrect, the response does not earn the second point.
- A response of $(3-0)\cdot C(0) + (7-3)\cdot C(3) + (12-7)\cdot C(7)$ earns the first point. Values must be pulled from the table to earn the second point.
- A response of $3\cdot 100 + 4\cdot 85 + 5\cdot 69$ earns both the first and second points, unless there is a subsequent error in simplification, in which case the response would earn only the first point.
- A completely correct right Riemann sum (e.g., $3\cdot 85 + 4\cdot 69 + 5\cdot 55$) earns 1 of the first 2 points. An unsupported answer of 806 does not earn either of the first 2 points.
- Units will not affect scoring for the second point.
- To earn the third point the interpretation must include both "average temperature" and the time interval. The response need not include a reference to units. However, if incorrect units are given in the interpretation, the response does not earn the third point.

Total for part (b) 3 points

(c) For $12 \le t \le 20$, the rate of change of the temperature of the coffee is modeled by $C'(t) = \frac{-24.55e^{0.01t}}{t}$, where $C'(t)$ is measured in degrees Celsius per minute. Find the temperature of the coffee at time $t = 20$. Show the setup for your calculations.

$C(20) = C(12) + \int_{12}^{20} C'(t)\,dt$	Integral	**1 point**
	Uses initial condition	**1 point**
$= 55 - 14.670812 = 40.329188$ The temperature of the coffee at time $t = 20$ is 40.329 degrees Celsius.	Answer	**1 point**

Scoring notes:

- The first point is earned for a definite integral with integrand $C'(t)$. If the limits of integration are incorrect, the response does not earn the third point.
- A linkage error such as $C(20) = \int_{12}^{20} C'(t)\,dt = 55 - 14.670812$ or $\int_{12}^{20} C'(t)\,dt = -14.670812 = 40.329188$ earns the first 2 points but does not earn the third point.
- Missing differential (dt):
  - Unambiguous responses of $C(20) = C(12) + \int_{12}^{20} C'(t)$ or $C(20) = 55 + \int_{12}^{20} C'(t)$ earn the first 2 points and are eligible for the third point.
  - Ambiguous responses of $C(20) = \int_{12}^{20} C'(t) + C(12)$ or $C(20) = \int_{12}^{20} C'(t) + 55$ do not earn the first point, earn the second point, and earn the third point if the given numeric answer is correct. If there is no numeric answer given, these responses do not earn the third point.
- The second point is earned for adding $C(12)$ or 55 to a definite integral with a lower limit of 12, either symbolically or numerically.
- The third point is earned for an answer of $55 - 14.671$ or $-14.671 + 55$ with no additional simplification, provided there is some supporting work for these values.
- An answer of just 40.329 with no supporting work does not earn any points.

Total for part (c) 3 points

(d) For the model defined in part (c), it can be shown that $C''(t) = \dfrac{0.2455e^{0.01t}(100 - t)}{t^2}$. For $12 < t < 20$, determine whether the temperature of the coffee is changing at a decreasing rate or at an increasing rate. Give a reason for your answer.

Because $C''(t) > 0$ on the interval $12 < t < 20$, the rate of change in the temperature of the coffee, $C'(t)$, is increasing on this interval. That is, on the interval $12 < t < 20$, the temperature of the coffee is changing at an increasing rate.	Answer with reason	**1 point**

Scoring notes:

- This point is earned only for a correct answer with a correct reason that references the sign of the second derivative of C.
- A response that provides a reason based on the evaluation of $C''(t)$ at a single point does not earn this point.
- A response that uses ambiguous pronouns (such as “It is positive, so increasing”) does not earn this point.
- A response does not need to reference the interval $12 < t < 20$ to earn the point.

Total for part (d) 1 point

Total for question 1 9 points

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Answer QUESTION 1 parts (a) and (b) on this page.

t (minutes)	0	3	7	12
$C(t)$ (degrees Celsius)	100	85	69	55

Response for question 1(a)

$C'(5) = \frac{69-85}{7-3} = -4$ degrees Celsius per minute

$\frac{f(b)-f(a)}{b-a}$

Response for question 1(b)

$3(100)+4(85)+5(69) = 985$ degrees Celsius

$\frac{1}{12}\int_0^{12} C(t)\,dt$ is the average temperature over the interval from 0 min to 12 min

1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1

Answer QUESTION 1 parts (c) and (d) on this page.

Response for question 1(c)

$55 + \int_{12}^{20} C`(t)\,dt = 40.329$ degrees Celsius

Response for question 1(d)

The temp of the coffee is changing at an increasing rate because C``(t) is + over the interval 12<t<20

1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1

Answer QUESTION 1 parts (a) and (b) on this page.

t (minutes)	0	3	7	12
$C(t)$ (degrees Celsius)	100	85	69	55

Response for question 1(a)

$$\frac{f(b)-f(a)}{b-a} = \frac{69-85}{7-3} = \frac{-16}{4} = -4$$

MVT $\frac{7+3}{2} = 10$ $\quad C'(5) \approx -4$ degrees C°/minute

Response for question 1(b)

$\int_0^{12} c(t)\,dt$

$\approx 3(100)+4(85)+5(69)$

≈ 985

$\frac{1}{12}\int_0^{12} c(t)\,dt$

$\approx 82.08333333\ ^\circ C$

This is the average temperature of the coffee from 0 to 12 minutes

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Q5223/4

Sample 1B

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Answer QUESTION 1 parts (c) and (d) on this page.

Response for question 1(c)

$c'(t) = \frac{-24.55e^{0.01t}}{t}$

$\int_{12}^{20} \frac{-24.55e^{0.01t}}{t}$

-14.67081194

Response for question 1(d)

$c''(t) = \frac{0.2455e^{0.01t}(100-t)}{t^2}$

$c''(16) = \frac{0.2455e^{0.01(16)}(100-16)}{(16)^2}$

$= 0.0945318015$

Increasing rate since $c''(16)$ is positive and in the given interval.

1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1

Answer QUESTION 1 parts (a) and (b) on this page.

t (minutes)	0	3	7	12
$C(t)$ (degrees Celsius)	100	85	69	55

Response for question 1(a)

$\frac{69-85}{7-3} = \frac{-16}{4} = -4$ degrees celchs

Response for question 1(b)

$(3\cdot100)+(4\cdot85)+(5\cdot69)$

$300+340+345$

985

$\frac{1}{12}\int_0^{12} C(t)$ is the average rate of changes for degrees celchs over 12 minutes of time

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Q5224/4

1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1

Answer QUESTION 1 parts (c) and (d) on this page.

Response for question 1(c)

$C'(20) = \frac{-24.55e^{0.01(20)}}{20}$

≈ -1.499 cokers

Response for question 1(d)

C''(t) is changing at an increasing rate as the equation $\int_{12}^{20} C''(t)\,dt$ ends up as a positive rate

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

0123122

5224/5

Question 1

Note: Student samples are quoted verbatim and may contain spelling and grammatical errors.

Overview

In this question students were given a table of times t in minutes, $0 \le t \le 12,$ and values of a decreasing differentiable function $C(t)$ that models the temperature, in degrees Celsius, of coffee in a cup.

In part (a) students were asked to approximate the value of $C'(5)$ using the average rate of change of C over the interval $3 \le t \le 7$ and to include correct units with their answer. A correct response will use values from the given table to calculate $C'(5) \approx \dfrac{C(7) - C(3)}{7 - 3} = \dfrac{69 - 85}{4} = -4$ degrees Celsius per minute.

In part (b) students were asked to approximate the value of $\int_0^{12} C(t)\,dt$ using a left Riemann sum with the three subintervals indicated by the values in the given table. Then students were asked to interpret the meaning of $\frac{1}{12}\int_0^{12} C(t)\,dt$ in the context of the problem. A correct response would present the left Riemann Sum setup and the approximation (e.g., $(3 - 0)\cdot C(0) + (7 - 3)\cdot C(3) + (12 - 7)\cdot C(7) = 985$). A correct response would also indicate that $\frac{1}{12}\int_0^{12} C(t)\,dt$ represents the average temperature of the coffee, in degrees Celsius, over the time interval from $t = 0$ to $t = 12.$

In part (c) $C'(t) = \dfrac{-24.55e^{0.01t}}{t}$ was introduced as a function that models the rate of change of the coffee's temperature, in degrees Celsius per minute, over the time interval $12 \le t \le 20$ Students were asked to find the temperature of the coffee at time $t = 20.$ A correct response would provide the setup $C(20) = C(12) + \int_{12}^{20} C'(t)\,dt$ then use a calculator to add the value $C(12) = 55$ to the value of integral and report a temperature of 40.329 degrees Celsius.

In part (d) students were given $C''(t) = \dfrac{0.2455e^{0.01t}(100 - t)}{t^2},$ the derivative of the model introduced in part (c), and asked to determine whether the temperature of the coffee was changing at a decreasing rate or at an increasing rate for $12 < t < 20.$ A correct response would observe that the given function, $C''(t),$ is positive on the interval $12 < t < 20,$ and therefore the rate of change of the temperature of the coffee, $C'(t),$ is increasing on this interval.

Question 1 (continued)

Sample: 1A
Score: 9

The response earned 9 points: 2 points in part (a), 3 points in part (b), 3 points in part (c), and 1 point in part (d).

In part (a) the response would have earned the first point in line 1 with the expression $\frac{69-85}{7-3}$ with no simplification. Because no simplification error is made, the response earned the first point. The response earned the second point for the correct units, "degrees Celsius per minute."

In part (b) the response earned the first point in line 1 with $3(100)+4(85)+5(69)$ and would have earned the second point with no simplification. Because no simplification error is made, the response earned the second point. The units reported in line 1, "degrees Celsius," although incorrect, do not affect scoring. The response earned the third point for "the average temperature over the interval from 0 min to 12 min" in lines 2 and 3.

In part (c) the response earned the first point with $\int_{12}^{20} C'(t)\,dt$. The response earned the second point with the sum of 55 and the definite integral. The response earned the third point with the correct answer of 40.329.

In part (d) the response earned the point with the correct conclusion "The temp of the coffee is changing at an increasing rate" and the statement "$C''(t)$ is $+$."".

Sample: 1B
Score: 6

The response earned 6 points: 2 points in part (a), 3 points in part (b), 1 point in part (c), and no points in part (d).

In part (a) the response would have earned the first point in line 1 with the expression $\frac{69-85}{7-3}$ with no simplification. Because no simplification error is made, the response earned the first point. The response earned the second point because of the correct units in line 2.

In part (b) the response earned the first point in line 2 with $3(100)+4(85)+5(69)$ and would have earned the second point with no simplification. Because no simplification error is made, the response earned the second point. The third point was earned for the phrase "average temperature of the coffee from 0 to 12 minutes" in lines 6 and 7.

In part (c) the response earned the first point with the definite integral that appears in line 2. The response did not earn the second point because the initial condition is never used. The response did not earn the third point because the stated answer is incorrect.

In part (d) the response did not earn the point because the reasoning is based on the evaluation of $C''(t)$ at a single point.

Question 1 (continued)

Sample: 1C
Score: 3

The response earned 3 points: 1 point in part (a), 2 points in part (b), no points in part (c), and no points in part (d).

In part (a) the response would have earned the first point in line 1 with the expression $\frac{69-85}{7-3}$ with no simplification. Because no simplification error is made, the response earned the first point. The second point is not earned because the units "degrees Celsius" are incorrect.

In part (b) the response earned the first point in line 1 on the left with $(3\cdot 100)+(4\cdot 85)+(5\cdot 69)$ and would have earned the second point with no simplification. Because no simplification error is made, the response earned the second point in line 3. The third point is not earned because the interpretation includes the phrase "average rate of change.".

In part (c) the response earned no points because no definite integral is presented.

In part (d) the response did not earn the point because the response states "$C''(t)$ is changing at an increasing rate" instead of $C(t)$.

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AP® Calculus BC

Sample Student Responses and Scoring Commentary

Inside:

Free-Response Question 2

- ☑ Scoring Guidelines
- ☑ Student Samples
- ☑ Scoring Commentary

Part A (BC): Graphing calculator required

Question 2 — 9 points

General Scoring Notes

The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

A particle moving along a curve in the xy-plane has position $(x(t), y(t))$ at time t seconds, where $x(t)$ and $y(t)$ are measured in centimeters. It is known that $x'(t) = 8t - t^2$ and $y'(t) = -t + \sqrt{t^{1.2} + 20}$. At time $t = 2$ seconds, the particle is at the point $(3, 6)$.

	Model Solution	Scoring	
(a)	Find the speed of the particle at time $t = 2$ seconds. Show the setup for your calculations.		
	$\sqrt{(x'(2))^2 + (y'(2))^2}$	Setup for speed	**1 point**
	$= 12.3048506$ The speed of the particle at time $t = 2$ seconds is 12.305 (or 12.304) centimeters per second.	Answer	**1 point**

Scoring notes:

- The first point is earned for the expression $\sqrt{(x'(2))^2 + (y'(2))^2}$, $\sqrt{(x'(t))^2 + (y'(t))^2}$, or equivalent.
- A response that presents just the exact answer, $\sqrt{144 + \left(-2 + \sqrt{2^{1.2} + 20}\right)^2}$, earns both points.
- The second point is earned only for the answer 12.305 (or 12.304) regardless of whether the first point is earned.
- A response that includes a linkage error, such as $\sqrt{(x'(t))^2 + (y'(t))^2} = \sqrt{144 + \left(-2 + \sqrt{2^{1.2} + 20}\right)^2}$ or $\sqrt{(x'(t))^2 + (y'(t))^2} = 12.305$ (or 12.304), earns at most 1 of the 2 points.
- Missing or incorrect units will not affect scoring in this part.

Total for part (a) — 2 points

(b) Find the total distance traveled by the particle over the time interval $0 \le t \le 2$. Show the setup for your calculations.

$\int_0^2 \sqrt{(x'(t))^2 + (y'(t))^2}\, dt$	Integral	**1 point**
$= 15.901715$ The total distance traveled by the particle over the time interval $0 \le t \le 2$ is 15.902 (or 15.901) centimeters.	Answer	**1 point**

Scoring notes:

- The first point is earned only for an integral of $\int_0^2 \sqrt{(x'(t))^2 + (y'(t))^2}\, dt$ (or the mathematical equivalent), with or without the differential.
  - Note: $\int_0^2 \sqrt{x'(t)^2 + y'(t)^2}\, dt$ is not read as a parenthesis error.
- The second point is earned only for an answer of 15.902 (or 15.901), regardless of whether the first point is earned.
- Missing or incorrect units will not affect scoring in this part.

Total for part (b) 2 points

(c) Find the y-coordinate of the position of the particle at the time $t = 0$. Show the setup for your calculations.

$y(0) = 6 + \int_2^0 y'(t)\, dt = 6 - 7.173613 = -1.173613$ The y-coordinate of the position of the particle at time $t = 0$ is -1.174 (or -1.173).	Definite integral	**1 point**
	Uses initial condition	**1 point**
	Answer	**1 point**

Scoring notes:

- An answer of -1.174 (or -1.173) with no supporting work does not earn any points.
- The first point is earned for either of the definite integrals $\int_2^0 y'(t)\, dt$ or $\int_0^2 y'(t)\, dt$.
- The second point is earned for any of:
  - $y(2) \pm \int_2^0 y'(t)\, dt,\ 6 \pm \int_2^0 y'(t)\, dt,$
  - $y(2) \pm \int_0^2 y'(t)\, dt,\ 6 \pm \int_0^2 y'(t)\, dt,$
  - $y(2) \pm 7.173613,$ or $6 \pm 7.173613.$
- A response that attempts to evaluate $\int y'(t)\, dt$ does not earn the first or the third point.
  - Such a response can earn the second point by attempting to solve for the constant of integration by presenting an expression as an antiderivative for $y'(t)$, evaluating this expression at $t = 2$, and setting this expression equal to 6.

- A response that reverses the limits of integration, e.g., $y(0) = 6 + \int_0^2 y'(t)\,dt$ or $6 + 7.173613,$ earns the second point but does not earn the third point.
- In order to earn the third point, a response must have earned at least 1 of the first 2 points.
- A response containing any linkage error can earn at most 2 of the 3 points. For example:
  - Equating two unequal quantities: $\int_2^0 y'(t)\,dt = -1.174,$ $\int_2^0 y'(t)\,dt = 6 - 7.173613,$ $6 + \int_0^2 y'(t)\,dt = 6 - 7.173613,$ or $6 + \int_0^2 y'(t)\,dt = 6 + 7.173613 = -1.173613$
  - Equating an expression to a numerical value: $y(t) = 6 + \int_2^0 y'(t)\,dt = -1.174$
- Missing differentials (dt):
  - Unambiguous responses of $y(2) + \int_2^0 y'(t),$ $y(2) - \int_0^2 y'(t),$ $6 + \int_2^0 y'(t),$ or $6 - \int_0^2 y'(t)$ earn the first 2 points and would earn the third point for the correct numerical answer.
  - Unambiguous responses of $y(2) + \int_0^2 y'(t)$ or $6 + \int_0^2 y'(t)$ with reversed limits of integration and missing differential earn the first 2 points but cannot earn the third point.
  - Ambiguous responses of $\int_2^0 y'(t) + y(2),$ $-\int_0^2 y'(t) + y(2),$ $\int_2^0 y'(t) + 6,$ or $-\int_0^2 y'(t) + 6$ earn the first point, do not earn the second point, but do earn the third point if a correct numeric answer is provided. If no numeric answer is given, none of these responses earn the third point.
  - Ambiguous responses of $\int_0^2 y'(t) + y(2)$ or $\int_0^2 y'(t) + 6$ with reversed limits of integration and no differential earn 1 out of 3 points.
- If a response provides work for both the x- and y-coordinates, the work for the x-coordinate will not affect scoring.
- However, a response that reports only a completely correct x-coordinate of the particle's position at time $t = 0$ with all supporting work, e.g., $x(0) = 3 + \int_2^0 x'(t)\,dt = -\frac{31}{3} = -10.333,$ earns 2 out of 3 points.

Total for part (c) 3 points

(d) For $2 \le t \le 8,$ the particle remains in the first quadrant. Find all times t in the interval $2 \le t \le 8$ when the particle is moving toward the x-axis. Give a reason for your answer.

Because $y(t) > 0$ when $2 \le t \le 8,$ the particle will be moving toward the x-axis when $y'(t) < 0.$ This occurs when 5.222 (or $5.221) < t < 8.$	Considers sign of $y'(t)$	**1 point**
	Answer with reason	**1 point**

Scoring notes:

- The first point can be earned by stating $y'(t) = 0,$ $y'(t) < 0,$ $y'(t) > 0,$ or $t = 5.222$ (or 5.221).

  Note: $y'(t)$ may be written as $\frac{dy}{dt}.$
- The second point cannot be earned without the first.
- To earn the second point, a response must identify the correct interval (and no additional intervals in $[2, 8]$) and explicitly state the need for $y'(t) < 0.$ The interval can be open, closed, or half-open.

Total for part (d) 2 points

Total for question 2 9 points

Sample 2A

Answer QUESTION 2 parts (a) and (b) on this page.

Response for question 2(a)

$x'(2) = 8(2) - 2^2 = 12$

$y'(2) = -2 + \sqrt{2^{1.2} + 20} = 2.7220$

speed: $\sqrt{(x'(2))^2 + (y'(2))^2} = \sqrt{12^2 + 2.7220^2} = 12.3049 \frac{cm}{sec}$

Response for question 2(b)

$\int_0^2 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = 15.9017 \text{ cm}$

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

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Answer QUESTION 2 parts (c) and (d) on this page.

Response for question 2(c)

$y(2) = 6$

$y(2) = y(0) + \int_0^2 y'(t)\,dt$

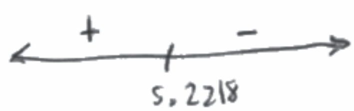
$6 = y(0) + 7.1736$

$y(0) = -1.1736$

Response for question 2(d)

moving toward x axis means $y'(t)$ is negative

$y'(t) = 0$ at $t = 5.2218$



The particle is moving towards the x-axis when $y'(t)$ is negative, which occurs when $5.2218 < t \leq 8$.

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

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Q5229/7

Sample 2B

2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2

Answer QUESTION 2 parts (a) and (b) on this page.

Response for question 2(a)

a.

$\int_0^2$

$\sqrt{(x'(2))^2 + (y'(2))^2} \approx 12.896$ cm/s

Response for question 2(b)

$\int_0^2 x'(t)\,dt = \frac{40}{3}$

$\int_0^2 y'(t)\,dt \approx 7.173613$

$\sqrt{\left(\frac{40}{3}\right)^2 + (7.173613)^2} \approx 15.141$ cm

Answer QUESTION 2 parts (c) and (d) on this page.

Response for question 2(c)

$6 - \int_0^2 (-t + \sqrt{t^{1.2} + 20})\,dt \approx -1.174$

Response for question 2(d)

$y'(t)$ = vertical velocity of particle

$y'(t) < 0 \quad [5.222, 8]$

when $y'(t)$ is negative, the particle is moving down and toward x-axis. There fore, prticle is moving toward x-axis $5.222 < t \leq 8$.

2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2

Answer QUESTION 2 parts (a) and (b) on this page.

Response for question 2(a)

$x'(2) = 8(2) - (2)^2$
$= 16 - 4$
$= 12$

$y'(2) = -2 + \sqrt{2^{1.2} + 20}$
≈ 2.722

Speed $= \sqrt{(x'(t))^2 + (y'(t))^2}$
$= \sqrt{(x'(2))^2 + (y'(2))^2}$
$= \sqrt{(12)^2 + (2.722)^2}$
$= \boxed{12.305}$

Response for question 2(b)

$TDT = \int_a^b |v(t)|\, dt$

$= \int_0^2 \left| \frac{x'(t)}{y'(t)} \right| dt$

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2

Answer QUESTION 2 parts (c) and (d) on this page.

Response for question 2(c)

$\int y'(t)\,dt = y(t)$

$y(t) = \frac{1}{2}t^2 +$

$(t^{1.2}+2t)^{1/2}$

$\frac{2}{3}(t^{1.2}+2t)^{3/2}$

Response for question 2(d)

The particle approaches the x-axis when $y'(t) < 0$ in the first quadrant.

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

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Q5228/7

Question 2

Note: Student samples are quoted verbatim and may contain spelling and grammatical errors.

Overview

In this question students were told a particle was moving along a curve in the xy-plane with position $(x(t), y(t))$ at time t seconds. Students were also given expressions for $x'(t)$ and $y'(t)$ and were told that the particle was at the point $(3, 6)$ at time $t = 2$.

In part (a) students were asked to find the speed of the particle at time $t = 2$ seconds. A correct response would provide the setup $\sqrt{(x'(2))^2 + (y'(2))^2}$ and use a calculator to find the numerical value 12.305 (or 12.304).

In part (b) students were asked to find the total distance traveled by the particle over the time interval $0 \le t \le 2$. A correct response would present the setup $\int_0^2 \sqrt{(x'(t))^2 + (y'(t))^2}\, dt$ and use a calculator to find the value 15.902 (or 15.901).

In part (c) students were asked to find the y-coordinate of the position of the particle at time $t = 0$. A correct response would provide the setup $y(0) = 6 + \int_2^0 y'(t)\, dt$ and use a calculator to find the value -1.174 (or -1.173).

In part (d) students were told that the particle remains in the first quadrant during the time interval $2 \le t \le 8$ and were asked to find all times in this interval when the particle is moving toward the x-axis. A correct response would consider the sign of $y'(t)$ and answer that because $y'(t) < 0$ on this time interval, the particle will be moving toward the x-axis.

Sample: 2A
Score: 9

The response earned 9 points:2 points in part (a), 2 points in part (b), 3 points in part (c), and 2 points in part (d).

In part (a) the response earned the first point with the expression $\sqrt{(x'(2))^2 + (y'(2))^2}$ on line 3. The response earned the second point with the expression $\sqrt{12^2 + 2.7220^2}$ with no subsequent errors in simplification. Note that the expression $\sqrt{12^2 + 2.7220^2}$ alone would have earned both points. The units, though correct, are not required to earn the second point.

In part (b) the response earned the first point with the definite integral $\int_0^2 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$. The response earned the second point with the boxed value of 15.9017. The units, although correct, are not required to earn the second point.

In part (c) the response earned the first point for the definite integral $\int_0^2 y'(t)\, dt$ in line 2. The response earned the second point for using the initial condition in the equation $y(2) = y(0) + \int_0^2 y'(t)\, dt$ in line 2. The response earned the third point with the boxed statement $y(0) = -1.1736$ in line 4.

Question 2 (continued)

In part (d) the response earned the first point for the statement " $y'(t)$ is negative" at the end of line 1, and is eligible to earn the second point. The response earned the second point for the correct interval with the reason "when $y'(t)$ is negative" given in the paragraph at the bottom right. The second point may be earned with a half open interval, as given in this response.

Sample: 2B
Score: 6

The response earned 6 points: 1 point in part (a), no points in part (b), 3 points in part (c), and 2 points in part (d).

In part (a) the response earned the first point with the correct expression for speed at time $t = 2$, $\sqrt{(x'(2))^2 + (y'(2))^2}$. The response presents an incorrect value for the speed at time $t = 2$ and did not earn the second point.

In part (b) the response does not present the correct definite integral and does not present the correct answer, thus did not earn either point.

In part (c) the response earned the first point with the definite integral $\int_0^2 \left(-t + \sqrt{t^{1.2} + 20}\right) dt$. The response earned the second point with the expression $6 - \int_0^2 \left(-t + \sqrt{t^{1.2} + 20}\right) dt$, demonstrating use of the initial condition. The response earned the third point with the presentation of the correct value -1.174.

In part (d) the response earned the first point with the statement $y'(t) < 0$ in line 2, and is eligible to earn the second point. The response earned the second point with the correct interval $(5.222, 8]$ and reason $y'(t) < 0$ both given in line 2, and no subsequent incorrect statements.

Sample: 2C
Score: 2

The response earned 2 points: 1 point in part (a), no points in part (b), no points in part (c), and 1 point in part (d).

In part (a) the response earned the first point in line 1 for the expression $\sqrt{(x'(t))^2 + (y'(t))^2}$. The response did not earn the second point. The response has a linkage error between line 1 and line 2, thus is eligible to earn at most 1 of the 2 points.

In part (b) the response did not earn the first point because the response presents a definite integral with an incorrect integrand in line 2. The response does not present the correct value for distance, and did not earn the second point.

In part (c) the response does not include a definite integral, and did not earn the first point. The second point was not earned because, although the response attempts to present an expression for the antiderivative of $y'(t)$ in the last line, the response does not include a constant of integration. Without an attempt to solve for the constant of integration at $t = 2$, the response cannot earn the second point. Neither of the first two points has been earned, so the response is not eligible to earn the third point.

In part (d) the response earned the first point for the statement $y'(t) < 0$ in line 1. The response did not earn the second point because the correct interval is not presented.

2024

AP® Calculus BC

Sample Student Responses and Scoring Commentary

Inside:

Free-Response Question 3

☑ Scoring Guidelines

☑ Student Samples

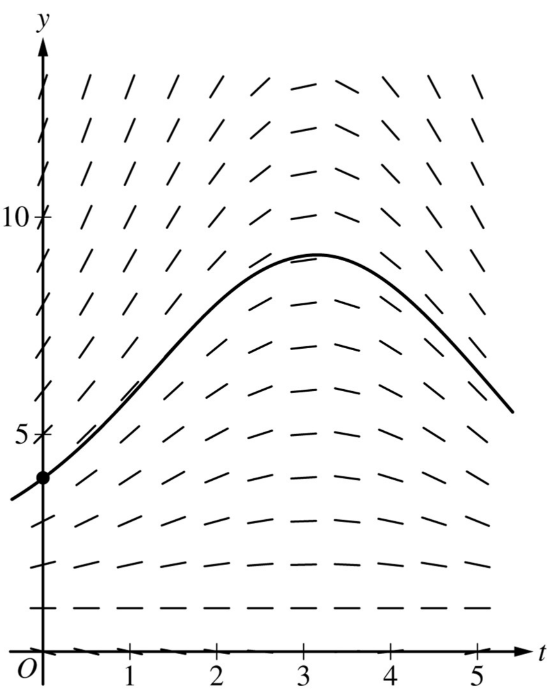
☑ Scoring Commentary

Part B (AB or BC): Graphing calculator not allowed
Question 3 — 9 points

General Scoring Notes
The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

The depth of seawater at a location can be modeled by the function H that satisfies the differential equation $\frac{dH}{dt} = \frac{1}{2}(H-1)\cos\left(\frac{t}{2}\right)$, where $H(t)$ is measured in feet and t is measured in hours after noon ($t = 0$). It is known that $H(0) = 4$.

	Model Solution	Scoring	
(a)	A portion of the slope field for the differential equation is provided. Sketch the solution curve, $y = H(t)$, through the point $(0, 4)$.		
		Solution curve	**1 point**

Scoring notes:

- The solution curve must pass through the point $(0, 4)$, extend to at least $t = 4.5$, and have no obvious conflicts with the given slope lines.
- Only portions of the solution curve within the given slope field are considered.

Total for part (a) — 1 point

(b) For $0 < t < 5,$ it can be shown that $H(t) > 1.$ Find the value of $t,$ for $0 < t < 5,$ at which H has a critical point. Determine whether the critical point corresponds to a relative minimum, a relative maximum, or neither a relative minimum nor a relative maximum of the depth of seawater at the location. Justify your answer.

Because $H(t) > 1,$ then $\frac{dH}{dt} = 0$ implies $\cos\left(\frac{t}{2}\right) = 0.$	Considers sign of $\frac{dH}{dt}$	**1 point**
This implies that $t = \pi$ is a critical point.	Identifies $t = \pi$	**1 point**
For $0 < t < \pi,$ $\frac{dH}{dt} > 0$ and for $\pi < t < 5,$ $\frac{dH}{dt} < 0.$ Therefore, $t = \pi$ is the location of a relative maximum value of $H.$	Answer with justification	**1 point**

Scoring notes:

- The first point is earned for considering $\frac{dH}{dt} = 0,$ $\frac{dH}{dt} > 0,$ $\frac{dH}{dt} < 0,$ $\cos\left(\frac{t}{2}\right) = 0,$ $\cos\left(\frac{t}{2}\right) > 0,$ or $\cos\left(\frac{t}{2}\right) < 0.$
- The second point is earned for identifying $t = \pi,$ with or without supporting work. A response may consider $H = 1$ or $t = 1$ as potential critical points without penalty.
- The third point cannot be earned without the first point. The third point is earned only for a correct justification and a correct answer of "relative maximum."
- The justification can be shown by determining the sign of $\frac{dH}{dt}$ (or $\cos\left(\frac{t}{2}\right)$) at a single value in $0 < t < \pi$ and at a single value in $\pi < t < 5.$ It is not necessary to state that $\frac{dH}{dt}$ does not change sign on these intervals.
- The third point can also be earned by using the Second Derivative Test. For example:

$$\frac{d^2H}{dt^2} = \frac{1}{2}(H-1)\left(-\frac{1}{2}\sin\left(\frac{t}{2}\right)\right) + \cos\left(\frac{t}{2}\right)\cdot\frac{1}{2}\cdot\frac{dH}{dt}$$

$$\left.\frac{d^2H}{dt^2}\right|_{t=\pi} < 0$$

Therefore, $t = \pi$ is the location of a relative maximum value of $H.$

Total for part (b) 3 points

(c) Use separation of variables to find $y = H(t)$, the particular solution to the differential equation $\frac{dH}{dt} = \frac{1}{2}(H-1)\cos\left(\frac{t}{2}\right)$ with initial condition $H(0) = 4$.

$\frac{dH}{H-1} = \frac{1}{2}\cos\left(\frac{t}{2}\right)dt$	Separation of variables	**1 point**
$\int \frac{dH}{H-1} = \int \frac{1}{2}\cos\left(\frac{t}{2}\right)dt$ $\Rightarrow \ln\lvert H-1\rvert = \sin\left(\frac{t}{2}\right) + C$	One antiderivative	**1 point**
	Second antiderivative	**1 point**
$\ln\lvert 4-1\rvert = \sin\left(\frac{0}{2}\right) + C \Rightarrow C = \ln 3$ Because $H(0) = 4$, $H > 1$, so $\lvert H-1\rvert = H-1$. $\ln(H-1) = \sin\left(\frac{t}{2}\right) + \ln 3$	Constant of integration and uses initial condition	**1 point**
$H - 1 = e^{\sin(t/2)+\ln 3} = 3e^{\sin(t/2)}$ $H(t) = 1 + 3e^{\sin(t/2)}$	Solves for H	**1 point**

Scoring notes:

- A response with no separation of variables earns 0 out of 5 points.
- A response that presents $\int \frac{dH}{H-1} = \ln(H-1)$ without absolute value symbols earns that antiderivative point.
- A response with no constant of integration can earn at most the first 3 points.
- A response is eligible for the fourth point only if it has earned the first point and at least 1 of the 2 antiderivative points.
- An eligible response earns the fourth point by correctly including the constant of integration in an equation and substituting 0 for t and 4 for H.
- A response is eligible for the fifth point only if it has earned the first 4 points.
- A response earns the fifth point only for an answer of $H(t) = 1 + 3e^{\sin(t/2)}$ or a mathematically equivalent expression for $H(t)$ such as $H(t) = 1 + e^{\sin(t/2)+\ln 3}$.
- A response does not need to argue that $\lvert H-1\rvert = H-1$ in order to earn the fifth point.
- Special case: A response that presents an incorrect separation of variables of $\frac{1}{2}\cdot\frac{dH}{H-1} = \cos\left(\frac{t}{2}\right)dt$ does not earn the first point or the fifth point but is eligible for the 2 antiderivative points. If the response earns at least 1 of the 2 antiderivative points, then the response is eligible for the fourth point.

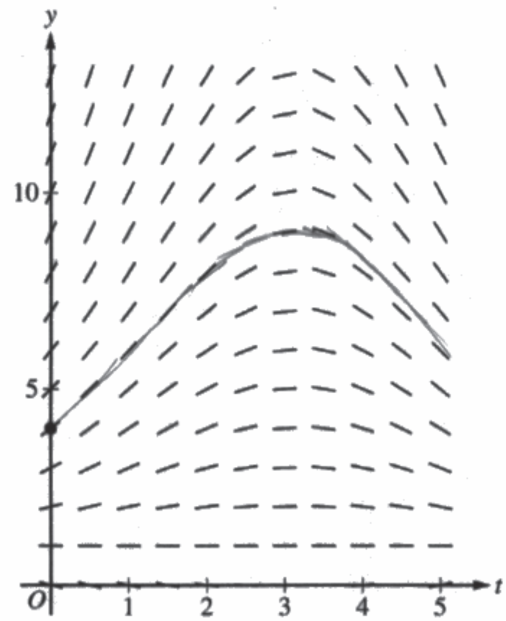
Total for part (c) 5 points

Total for question 3 9 points

3 3 3 3 3 NO CALCULATOR ALLOWED 3 3 3 3 3

Answer QUESTION 3 parts (a) and (b) on this page.

Response for question 3(a)



Response for question 3(b)

$H > 1$

$\frac{dH}{dt} = 0$

$0 = \frac{1}{2}(H-1)\cos\left(\frac{t}{2}\right)$

$0 = \cos\left(\frac{t}{2}\right)$

$t = \pi, 3\pi$

+ − ← π →

H has a critical point at π on the interval $0 < t < 5$, and the point is a relative maximum because H' changes from positive to negative at $t = \pi$.

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Q5224/8

Answer QUESTION 3 part (c) on this page.

Response for question 3(c)

$$\frac{dH}{dt} = \frac{1}{2}(H-1)\cos\left(\frac{t}{2}\right)$$

$$dH \cdot \frac{1}{\frac{1}{2}(H-1)} = \cos\left(\frac{t}{2}\right)dt$$

$$dH \cdot \frac{2}{H-1} = \cos\left(\frac{t}{2}\right)dt$$

$$\int \frac{2\,dH}{H-1} = \int \cos\left(\frac{t}{2}\right)dt$$

$$2\ln|H-1| = 2\sin\left(\frac{t}{2}\right) + C$$

$$2\ln|3| = 2\sin(0) + C$$

$$2\ln|3| = C$$

$$\ln|H-1| = \sin\left(\frac{t}{2}\right) + \ln|3|$$

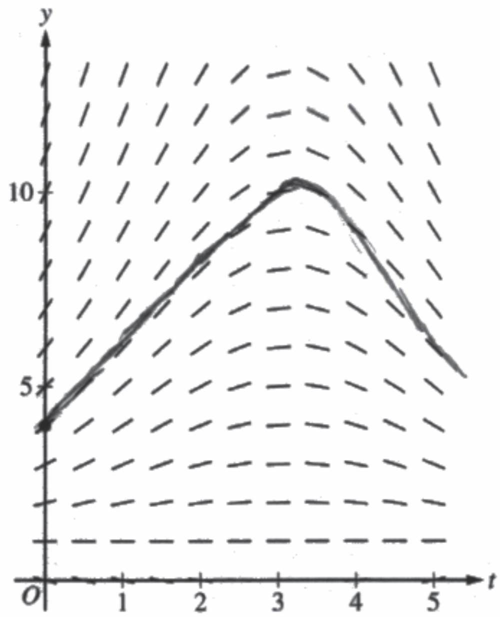
$$H-1 = e^{\sin\frac{t}{2} + \ln 3}$$

$$H = e^{\sin\frac{t}{2} + \ln 3} + 1$$

3 3 3 3 3 NO CALCULATOR ALLOWED 3 3 3 3 3

Answer QUESTION 3 parts (a) and (b) on this page.

Response for question 3(a)



Response for question 3(b)

$\frac{dH}{dt} = \frac{1}{2}(H-1)\cos\left(\frac{t}{2}\right)$

$\frac{dH}{dt} = \frac{1}{2}(H-1)\cos\left(\frac{t}{2}\right)$

$\frac{dH}{dt} = \frac{1}{2}(H-1)\cos(0)$

$\frac{dH}{dt} = \frac{3}{2}$

$0 = \frac{1}{2}(H-1)\cos\left(\frac{t}{2}\right)$

H has no critical point.

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Q5223/8

Answer QUESTION 3 part (c) on this page.

Response for question 3(c)

$$\frac{dH}{dt} = \frac{1}{2}(H-1)\cos\left(\frac{t}{2}\right)$$

$$\int \frac{1}{H-1}\,dH = \int \frac{1}{2}\cos\left(\frac{t}{2}\right)dt \qquad u = \frac{t}{2}$$

$$\int \frac{1}{H-1}\,dH = \frac{1}{2}\int \cos(u)\,\frac{du}{2} \qquad \frac{du}{2} = dt$$

$$\ln|H-1| = \frac{1}{4}\sin\left(\frac{t}{2}\right) + C$$

$$\ln|4-1| = \frac{1}{4}\sin(0) + C$$

$$\ln|3| = C$$

$$e^{(\ln|H-1|)} = e^{\left(\frac{1}{4}\sin\left(\frac{t}{2}\right) + \ln|3|\right)}$$

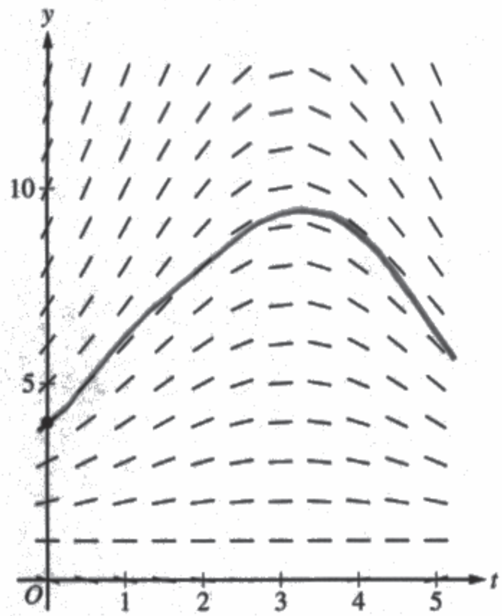
$$|H-1| = 3e^{\frac{1}{4}\sin\left(\frac{t}{2}\right)}$$

$$\boxed{H = 3e^{\frac{1}{4}\sin\left(\frac{t}{2}\right)} + 1}$$

3 3 3 3 3 NO CALCULATOR ALLOWED 3 3 3 3 3

Answer QUESTION 3 parts (a) and (b) on this page.

Response for question 3(a)



Response for question 3(b)

$\frac{dH}{dt}(H-1)\cos\left(\frac{t}{2}\right)$

$\frac{1}{2}(H-1)\cos\left(\frac{0}{2}\right)$

$\frac{1}{2}(H-1)\ 1$

$\frac{1}{2}(4-1)$

$\frac{1}{2}(3) = \frac{3}{2}$

$-\ |\ +$

$\frac{3}{2}$

$\frac{3}{2}$ is a relative minimum

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Q5223/8

3 3 3 3 3 NO CALCULATOR ALLOWED 3 3 3 3 3

Answer QUESTION 3 part (c) on this page.

Response for question 3(c)

$H(0)=4$

$\frac{dH}{dt} = \frac{1}{2}(H-1)\cos(\frac{t}{2})$

$\int \frac{dH}{(H-1)} = \int \frac{1}{2}\cos(\frac{t}{2})$

$\int \frac{1}{2}\sin(\frac{0}{2})$

$\sin(0)$

1

$\frac{1}{2}\cdot 1 + C$

$\ln|H-1| = \frac{1}{2} + C$

$\ln|0-1|$

$\ln|-1|$

$1 = \frac{1}{2} + C$

$1 - \frac{1}{2} + C = 4$

$\frac{2}{2} - \frac{1}{2}$

$= \frac{1}{2} + C = 4 - \frac{1}{2}$

$C = \frac{8}{2} - \frac{1}{2}$

$C = \frac{7}{2}$

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Q5223/9

Question 3

Note: Student samples are quoted verbatim and may contain spelling and grammatical errors.

Overview

In this question students were told that the depth of sea water, in feet, could be modeled by the function H which satisfies the differential equation $\frac{dH}{dt} = \frac{1}{2}(H-1)\cos\left(\frac{t}{2}\right)$, where t is measured in hours after noon. Furthermore, $H(0) = 4$, and so at noon the depth of the seawater is 4 feet.

In part (a) students were given a slope field for the differential equation and asked to sketch the solution curve through the point $(0, 4)$. A correct response will draw a curve that passes through the point $(0, 4)$, follows the indicated slope segments, and extends to at least $t = 4.5$.

In part (b) students were told that $H(t) > 1$ and asked to find the value of t in $0 < t < 5$ at which H has a critical point. Then the students were asked to determine whether this critical point is the location of a relative minimum, a relative maximum, or neither a relative minimum nor a relative maximum of the seawater depth. A correct response would first determine that $\left.\frac{dH}{dt}\right|_{t=\pi} = 0$ and therefore the critical point in $0 < t < 5$ occurs when $t = \pi$. Because $\frac{dH}{dt}$ changes from positive to negative at $t = \pi$, this critical point is the location of a relative maximum value of H.

In part (c) students were asked to use the separation of variables technique to find an expression for the particular solution to the differential equation $\frac{dH}{dt} = \frac{1}{2}(H-1)\cos\left(\frac{t}{2}\right)$ with initial condition $H(0) = 4$. A correct response will separate the variables H and t, integrate, use the initial condition to find the value of the constant of integration, and arrive at a solution of $H(t) = 1 + 3e^{\sin(t/2)}$.

Sample: 3A
Score: 9

The response earned 9 points: 1 point in part (a), 3 points in part (b), and 5 points in part (c).

In part (a) the response earned the point with a correct sketch of H.

In part (b) the response earned the first point with $\frac{dH}{dt} = 0$. The response earned the second point by identifying π in line 5. The response earned the third point with the statement "the point is a relative maximum because H' changes from positive to negative at $t = \pi$."

In part (c) the response earned the first point with an acceptable separation of variables in line 2. The second and third points are earned with correct antiderivatives in line 5. The fourth point is earned with the constant of integration appearing in line 5 and the initial condition being used correctly in line 6. The fifth point is earned with a mathematically equivalent expression for $H(t)$ in line 10.

Question 3 (continued)

Sample: 3B
Score: 5

The response earned 5 points: 1 point in part (a), 1 point in part (b), and 3 points in part (c).

In part (a) the response earned the point with a correct sketch of H.

In part (b) the response earned the first point with the equation $0 = \frac{1}{2}(H - 1)\cos\left(\frac{t}{2}\right)$ in line 1 on the right. The second and third points were not earned.

In part (c) the response earned the first point with a correct separation of variables in line 2. The second point was earned with a correct antiderivative on the left side of the equation in line 4. The third point was not earned because the antiderivative on the right side of the equation in line 4 is incorrect. The response is eligible for the fourth point. The fourth point was earned with the constant of integration appearing in line 4 and the initial condition used correctly in line 5. The response is not eligible for the fifth point.

Sample: 3C
Score: 3

The response earned 3 points: 1 point in part (a), no points in part (b), and 2 points in part (c).

In part (a) the response earned the point with a correct sketch of H.

In part (b) the first point was not earned because the response does not consider the sign of $\frac{dH}{dt}$. The second point was not earned because $t = \pi$ is not identified. The response is not eligible for the third point.

In part (c) the response earned the first point with an acceptable separation of variables in line 2. The second point was earned with a correct antiderivative on the left side of the equation in line 6. The third point was not earned because there is not a correct antiderivative for $\frac{1}{2}\cos\left(\frac{t}{2}\right)$. The fourth point was not earned because the initial condition is not used correctly: 0 is substituted for H instead of 4. The response is not eligible for the fifth point.

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Sample Student Responses and Scoring Commentary

Inside:

Free-Response Question 4

☑ Scoring Guidelines

☑ Student Samples

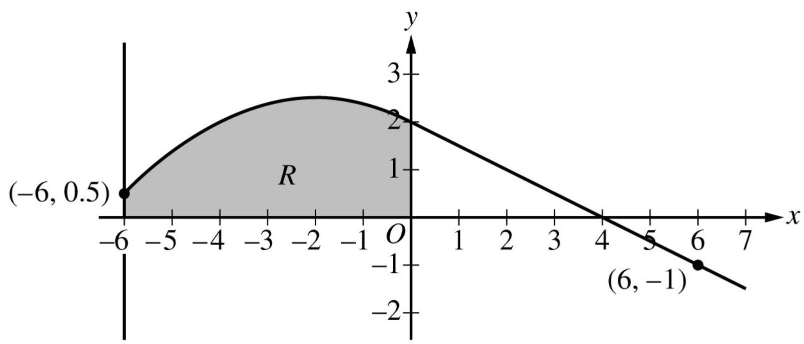
☑ Scoring Commentary

Part B (AB or BC): Graphing calculator not allowed
Question 4 — 9 points

General Scoring Notes

The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.



Graph of f

The graph of the differentiable function f, shown for $-6 \le x \le 7$, has a horizontal tangent at $x = -2$ and is linear for $0 \le x \le 7$. Let R be the region in the second quadrant bounded by the graph of f, the vertical line $x = -6$, and the x- and y-axes. Region R has area 12.

Model Solution	Scoring	
(a) The function g is defined by $g(x) = \int_0^x f(t)\,dt$. Find the values of $g(-6)$, $g(4)$, and $g(6)$.		
$g(-6) = \int_0^{-6} f(t)\,dt = -\int_{-6}^{0} f(t)\,dt = -12$	$g(-6)$	**1 point**
$g(4) = \int_0^4 f(t)\,dt = \frac{1}{2}\cdot 4\cdot 2 = 4$	$g(4)$	**1 point**
$g(6) = \int_0^6 f(t)\,dt = \frac{1}{2}\cdot 4\cdot 2 - \frac{1}{2}\cdot 2\cdot 1 = 3$	$g(6)$	**1 point**

Scoring notes:

- Supporting work is not required for any of these values. However, any supporting work that is shown must be correct to earn the corresponding point.
- Special case: A response that explicitly presents $g(x) = \int_{-6}^{x} f(t)\,dt$ does not earn the first point it would have otherwise earned. The response is eligible for all subsequent points for correct answers, or for consistent answers with supporting work.
    - Note: $\int_{-6}^{-6} f(t)\,dt = 0$, $\int_{-6}^{4} f(t)\,dt = 16$, $\int_{-6}^{6} f(t)\,dt = 15$
- Labeled values may be presented in any order. Unlabeled values are read from left to right and from top to bottom as $g(-6)$, $g(4)$, and $g(6)$, respectively. A response that presents only 1 or 2 values must label them to earn any points.

Total for part (a) 3 points

(b) For the function g defined in part (a), find all values of x in the interval $0 \le x \le 6$ at which the graph of g has a critical point. Give a reason for your answer.

$g'(x) = f(x)$	Fundamental Theorem of Calculus	**1 point**
$g'(x) = f(x) = 0 \Rightarrow x = 4$ Therefore, the graph of g has a critical point at $x = 4$.	Answer with reason	**1 point**

Scoring notes:

- The first point is earned for explicitly making the connection $g' = f$ in this part.
    - A response that writes $g'' = f'$ earns the first point but can only earn the second point by reasoning from $f = 0$.
- A response that does not earn the first point is eligible to earn the second point with an implied application of the FTC (e.g., "Because $g'(4) = 0$, $x = 4$ is a critical point").
- A response that reports any additional critical points in $0 < x < 6$ does not earn the second point.
    - Any presented critical point outside the interval $0 < x < 6$ will not affect scoring.

Total for part (b) 2 points

(c) The function h is defined by $h(x) = \int_{-6}^{x} f'(t)\,dt$. Find the values of $h(6)$, $h'(6)$, and $h''(6)$. Show the work that leads to your answers.

$h(6) = \int_{-6}^{6} f'(t)\,dt = f(6) - f(-6) = -1 - 0.5 = -1.5$	Uses Fundamental Theorem of Calculus	**1 point**
	$h(6)$ with supporting work	**1 point**
$h'(x) = f'(x)$, so $h'(6) = f'(6) = -\frac{1}{2}$.	$h'(6)$	**1 point**
$h''(x) = f''(x)$, so $h''(6) = f''(6) = 0$.	$h''(6)$	**1 point**

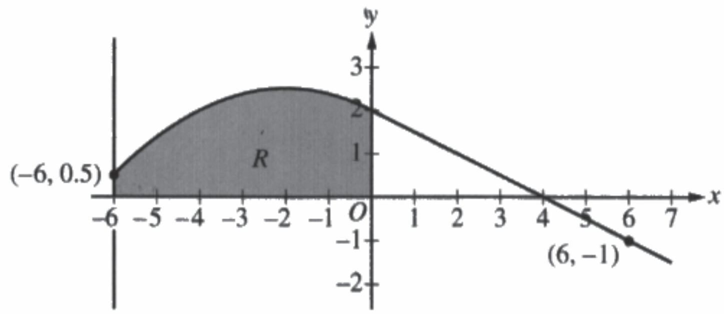
Scoring notes:

- Labeled values may be presented in any order.
- Unlabeled values are read from left to right and from top to bottom as $h(6)$, $h'(6)$, and $h''(6)$, respectively. A response that presents only 1 or 2 values must label them in order to earn any points.
- A response of $h(6) = -1.5$ does not earn either of the first 2 points. A response of $h(6) = f(6) - f(-6)$ earns the first point but not yet the second point.
- A response of $h(6) = -1 - 0.5$ is the minimum work required to earn both of the first 2 points.
- To earn the third point a response must state either $h'(x) = f'(x)$ or $h'(6) = f'(6)$, and provide an answer of $-\frac{1}{2}$.
- The fourth point is earned for a response of $h''(6) = 0$, with or without supporting work.
- A response that has one or more linkage errors does not earn the first point it would have otherwise earned. For example, $h'(x) = f'(6) = -\frac{1}{2}$ does not earn the third point but is eligible for the fourth point even in the presence of another linkage error, such as $h''(x) = f''(6) = 0$.

Total for part (c) 4 points

Total for question 4 9 points

Answer QUESTION 4 part (a) on this page.



Graph of f

Response for question 4(a)

$g(-6) = \int_0^{-6} f(t)\,dt = \boxed{-12}$

$g(4) = \int_0^{4} f(t)\,dt = \boxed{4}$

$g(6) = \int_0^{6} f(t)\,dt = \boxed{3}$

4 4 4 4 4 NO CALCULATOR ALLOWED 4 4 4 4 4

Answer QUESTION 4 parts (b) and (c) on this page.

Response for question 4(b)

$g'(x) = \frac{d}{dx}\int_0^x f(t)\,dt = f(x)$ fundamental Theorem of Calculus II

$g'(x) = f(x) = 0$

$\hookrightarrow x = \boxed{4}$

g has a critical point at x = 4 because g'(x), the slope of g(x), equals 0 at x = 4. We know this because g'(x) = f(x) by the Fundamental Theorem of Calculus II, & f(4) = 0.

Response for question 4(c)

$h(x) = \int_{-6}^x f'(t)\,dt = f(t)\Big|_{-6}^x$

$h(6) = \int_{-6}^6 f'(t)\,dt = f(t)\Big|_{-6}^6 = f(6) - f(-6) = -1 - 0.5 = \boxed{-1.5}$

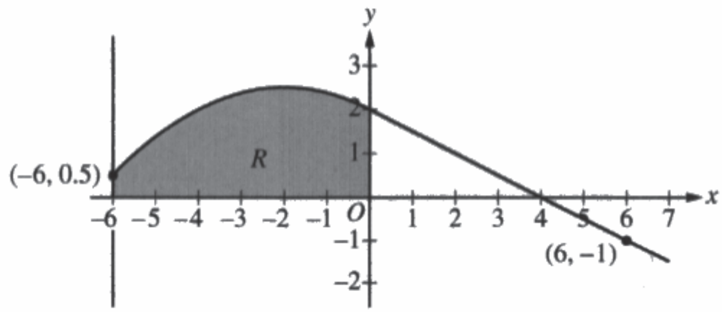
$h'(x) = \frac{d}{dx}\int_{-6}^x f'(t)\,dt = f'(x)$

$h'(6) = f'(6) = \boxed{-\frac{1}{2}}$

$h''(6) = f''(6) = \boxed{0}$

4 4 4 4 4 **NO CALCULATOR ALLOWED** 4 4 4 4 4

Answer QUESTION 4 part (a) on this page.



Graph of f

Response for question 4(a)

$g(-6) = \int_0^{-6} f(t) = \int_{-6}^{0} -f(x) \Rightarrow 12$

$g(4) = \int_0^{4} f(t) = \frac{1}{2}(4)(2) = 4$

$g(6) = \int_0^{6} f(t) = \int_0^{6} f(x) \Rightarrow 3$

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

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Q5229/10

Answer QUESTION 4 parts (b) and (c) on this page.

Response for question 4(b)

$g(x) = \int_0^x f(t)\,dt$

$g'(x) = f(t)$

$g'(x) = 0$ where $f(t) = 0$

at $t = 4$

$g(x)$ has a critical point at $x=4$ because $g'(t) = f(x)$ and $f(x) = 0$ so $g'(t) = 0$ at $x = 4$

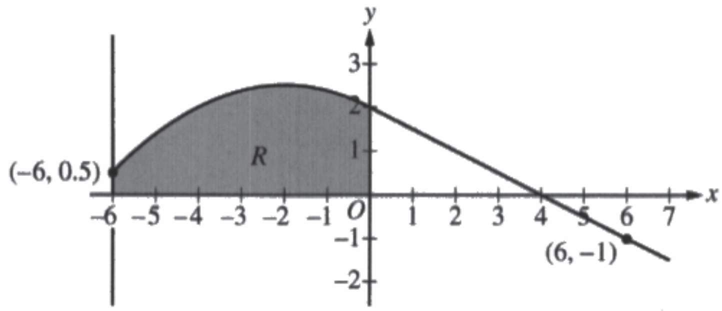
Response for question 4(c)

$h(6) = \int_{-6}^{6} f'(t) \Rightarrow f'(6) - f'(-6) \Rightarrow \frac{-1}{2} - 0 \Rightarrow \frac{-1}{2}$

$h'(6) = f'(6) = \frac{-1}{2}$

$h''(6) = f''(6) \Rightarrow 0$

Answer QUESTION 4 part (a) on this page.



Graph of f

Response for question 4(a)

$\int_0^{-6} f(t) = 12$

$\int_0^{4} f(t) = 4$

$\int_0^{6} f(t) = 4 + 1 = 5$

Answer QUESTION 4 parts (b) and (c) on this page.

Response for question 4(b)

$x = 4$

Critical points match inflections when taking derivative equation

Response for question 4(c)

$h'(6) = f'(6) - f'(-6)$

$-\frac{1}{2} - \frac{3}{8}$

$-\frac{7}{8}$

$h''(6) = f''(6) - f''(-6)$

$h(6) \int_{-6}^{6} f'(t)$

$f(6) - f(-6)$

$-1 - .5$

$-\frac{3}{2}$

Question 4

Note: Student samples are quoted verbatim and may contain spelling and grammatical errors.

Overview

In this question the graph of a differentiable function f, for $-6 \le x \le 7$, and the shaded region R in the second quadrant bounded by the graph of f, the vertical line $x = -6$ and the x- and y-axes are shown. Students are told that f has a horizontal tangent at $x = -2$ and is linear for $0 \le x \le 7$. Students are also told that region R has area 12.

In part (a) the function $g(x) = \int_0^x f(t)\,dt$, is defined and students are asked to find the values $g(-6)$, $g(4)$, and $g(6)$. A correct response would recognize that $g(-6) = \int_0^{-6} f(t)\,dt = -(\text{area of } R) = -12$. In addition, $g(4) = \int_0^4 f(t)\,dt$ is the area of a triangle of base 4 and height 2, so $g(4) = 4$. Finally, $g(6) = g(4) + \int_4^6 f(t)\,dt$, where is the area of a triangle with base 2 and height -1. Thus, $g(6) = 4 - 1 = 3$. Throughout this part of the problem, students are asked to demonstrate knowledge of the properties of definite integrals.

In part (b) students are asked to find all values of x in the interval $0 \le x \le 6$ at which the graph of g has a critical point. A correct response would recognize that by the Fundamental Theorem of Calculus, the derivative of the function g is the function f ($g' = f$), and therefore the critical points of g occur where $f(x) = 0$ or where $f(x)$ is undefined. Because $f(x)$ is differentiable, $f(x)$ is defined for all x in the interval $[0, 6]$. Therefore, $f(x) = 0 \Rightarrow x = 4$. Thus, the only critical point in this interval occurs at $x = 4$.

In part (c) a third function, h, is defined as $h(x) = \int_{-6}^x f'(t)\,dt$, and students are asked to evaluate this function and its first two derivatives at $x = 6$. A correct response will use the Fundamental Theorem of Calculus and the given graph of f to find that $h(6) = \int_{-6}^6 f'(t)\,dt = f(6) - f(-6) = -1.5$. Applying the Fundamental Theorem of Calculus again, a correct response would indicate that $h'(x) = f'(x)$ and use the fact that f is linear for $0 \le x \le 7$ to find that $h'(6) = f'(6)$ equals the slope of f at $x = 6$. Finally, a correct response would report that $h''(6) = f''(6) = 0$ because f is linear for $0 \le x \le 7$.

Sample: 4A
Score: 9

The response earned 9 points: 3 points in part (a), 2 points in part (b), and 4 points in part (c).

In part (a) the response earned the first point on the first line with the conclusion $g(-6) = -12$. The response earned the second point on the second line with the conclusion $g(4) = 4$. The response then earned the third point on the third line with the conclusion $g(6) = 3$. Although supporting work was not required to earn any of these three points, any supporting work presented must be correct. The response has all correct corresponding work with the presentation of integrals on each of the three lines.

In part (b) the response earned the first point on the first two lines with the explicit application of the Fundamental Theorem of Calculus (FTC), with correct supporting work, resulting with the conclusion that $g'(x) = f(x)$. The response earned the second point on the last three lines with the conclusion that "g has a critical point at $x = 4$

Question 4 (continued)

because $g'(x)$, the slope of $g(x)$, equals 0 at $x = 4$." The response continues correctly to refer to the fact that $g'(x) = f(x)$ by the FTC.

In part (c) the response earned the first point on the second line with the intermediate step indicating that $h(6) = f(6) - f(-6)$. The response earned the second point by correctly linking the above work with the numerical value $-1 - 0.5 = -1.5$. Note that the expression $-1 - 0.5$ appropriately linked to $h(6)$ would have earned both the first two points. The response earned the third point on the third line with the conclusion on the penultimate line that $h'(6) = -\frac{1}{2}$ with the supporting work on the line above indicating that $h'(x) = f'(x)$. The response earned the fourth point on the last line with the conclusion $h''(6) = 0$.

Sample: 4B
Score: 6

The response earned 6 points: 2 points in part (a), 2 points in part (b), and 2 points in part (c).

In part (a) the response did not earn the first point as the answer presented for $g(-6)$ at the end of the first line is 12, which is incorrect. The response would have earned the second and third points with the 4 and 3, presented at the ends of lines two and three, unsupported. Because support is presented, that support must be correct to earn each point. The supporting work on lines two and three is correct, thus the second and third points were earned.

In part (b) the response earned the first point with the equation on the second line. This is then reinforced on the third line with the indication that g' is 0 where f is 0. Note that although there is a difference between the independent variables presented with the above functions, this point is earned for correctly identifying the relationship between g and f via the FTC, as this response has done. The response earned the second point with the conclusion on the last two lines that "$g(x)$ has a critical point at $x = 4$" and the supporting reason that g' is 0 at $x = 4$.

In part (c) the response did not earn the first point as the presentation on the first line that $h(6)$ is $f'(6) - f'(-6)$ is incorrect. The response did not earn the second point as the value presented for $h(6)$ is incorrect. The response earned the third point on the penultimate line with the conclusion that $h'(6) = f'(6) = -\frac{1}{2}$. Note here that the intermediate step $h'(6) = f'(6)$ is necessary to earn the third point. It would have also sufficed to present $h'(x) = f'(x)$ to justify a correct evaluation of $h'(6)$. The response earned the fourth point with the correct conclusion that $h''(6)$ is 0.

Sample: 4C
Score: 3

The response earned 3 points: 1 point in part (a), no points in part (b), and 2 points in part (c).

In part (a) the response did not earn the first point as the first statement that $\int_0^{-6} f(t) = 12$ is not true. Note that as this response does not label the three answers, it is assumed that they are in the order presented in the question. The response earned the second point on the second line with the statement $\int_0^{4} f(t) = 4$. The response did not earn the third point as the answer of 5 presented is not correct.

Question 4 (continued)

In part (b) the response did not earn the first point as no connection is presented between functions f and g via the FTC. Although the response presents the correct location of $x = 4$ for a critical point, the reasoning is that "Critical points match inflections when taking derivative equation." is not correct.

In part (c) the response earned the first point on the second line in the work on the left, as the presented expression $f(6) - f(-6)$ provides evidence for the use of the FTC in evaluating $h(6)$. The response would have earned the second point on the third line on the left with the expression $-1 - .5$ without simplification. The simplification is correct, and the point was earned on the last line. The response did not earn the third point as the answer presented for $h'(6)$ is incorrect. The response did not earn the fourth point as no answer is presented for $h''(6)$.

2024

AP® Calculus BC

Sample Student Responses and Scoring Commentary

Inside:

Free-Response Question 5

☑ Scoring Guidelines

☑ Student Samples

☑ Scoring Commentary

Part B (BC): Graphing calculator not allowed
Question 5 — 9 points

General Scoring Notes
The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

x	0	π	2π
$f'(x)$	5	6	0

The function f is twice differentiable for all x with $f(0) = 0$. Values of f', the derivative of f, are given in the table for selected values of x.

	Model Solution	Scoring	
(a)	For $x \ge 0$, the function h is defined by $h(x) = \int_0^x \sqrt{1 + (f'(t))^2}\, dt$. Find the value of $h'(\pi)$. Show the work that leads to your answer.		
	$h'(x) = \sqrt{1 + (f'(x))^2}$	Fundamental Theorem of Calculus	**1 point**
	$h'(\pi) = \sqrt{1 + (f'(\pi))^2} = \sqrt{1 + 6^2} = \sqrt{37}$	Answer	**1 point**

Scoring notes:

- A response of $\sqrt{1 + (f'(\pi))^2}$ earns the first point.
- A response of $\sqrt{1 + 6^2}$ alone earns both points.
- A response such as $h'(x) = \sqrt{1 + (f'(x))^2} = \sqrt{37}$, that equates a variable expression to a numeric value, earns at most 1 of the 2 points.
- A response that equates $h'(x)$ or $h'(\pi)$ to a derivative of a constant, such as $h'(x) = \frac{d}{dx}\int_0^{\pi} \sqrt{1 + (f'(t))^2}\, dt$, earns at most 1 of the 2 points.

Total for part (a) 2 points

(b) What information does $\int_0^{\pi} \sqrt{1 + (f'(x))^2}\, dx$ provide about the graph of f?

$\int_0^{\pi} \sqrt{1 + (f'(x))^2}\, dx$ is the arc length of the graph of f on $[0, \pi]$.	Arc length of f	**1 point**
	Interval $[0, \pi]$	**1 point**

Scoring notes:

- A response of "arc length" or "length" earns the first point. Such a response does not need to reference f. However, if the response references a different function, the response does not earn the first point and is eligible to earn the second point.
- A response referring to distance explicitly connected to the graph or f (or equivalent) earns the first point. For example, a response of "distance along the curve" or "distance traveled by a particle moving along f" earns the first point and is eligible to earn the second point.
- A response referring to distance that is not explicitly connected to the graph of f does not earn the first point but is eligible to earn the second point. For example, a response of "distance" or "distance traveled" does not earn the first point but is eligible to earn the second point.
- To earn the second point a response must connect the interval $[0, \pi]$ to arc length, length, or distance.

Total for part (b) 2 points

(c) Use Euler's method, starting at $x = 0$ with two steps of equal size, to approximate $f(2\pi)$. Show the computations that lead to your answer.

$f(\pi) \approx f(0) + \pi f'(0) = 0 + 5\pi = 5\pi$ $f(2\pi) \approx f(\pi) + \pi f'(\pi)$	Euler's method	**1 point**
$\approx 5\pi + 6\pi = 11\pi$	Answer	**1 point**

Scoring notes:

- To earn the first point a response must demonstrate two Euler's steps, with use of the correct expression for $\frac{dy}{dx}$, and at most one error. If there is an error, the second point is not earned.
- In order to earn the first point, a response that presents a single error in computing the approximation of $f(\pi)$ must import the incorrect value in computing the approximation of $f(2\pi)$.
- The two Euler's steps may be explicit expressions or may be presented in a table. For example:

x	y	$\frac{dy}{dx} \cdot \Delta x$ (or $\frac{dy}{dx} \cdot \pi$)
0	0	5π
π	5π	6π
2π	11π	

  - In the presence of a correct answer, a table does not need to be labeled in order to earn both points. In the presence of no answer or an incorrect answer, such a table must be correctly labeled in order to earn the first point.
- Both points are earned for $5\pi + 6\pi$.
- The response may report the final answer as $(2\pi, 11\pi)$.

Total for part (c) 2 points

(d) Find $\int (t+5)\cos\left(\frac{t}{4}\right) dt$. Show the work that leads to your answer.

$u = t + 5 \qquad dv = \cos\left(\frac{t}{4}\right) dt$ $du = dt \qquad v = 4\sin\left(\frac{t}{4}\right)$	u and dv	**1 point**
$\int (t+5)\cos\left(\frac{t}{4}\right) dt = 4(t+5)\sin\left(\frac{t}{4}\right) - \int 4\sin\left(\frac{t}{4}\right) dt$	$uv - \int v\, du$	**1 point**
$= 4(t+5)\sin\left(\frac{t}{4}\right) + 16\cos\left(\frac{t}{4}\right) + C$	Answer	**1 point**

Scoring notes:

- The first and second points are earned with an implied u and dv in the presence of $4(t+5)\sin\left(\frac{t}{4}\right) - \int 4\sin\left(\frac{t}{4}\right) dt$ or a mathematically equivalent expression.
- The tabular method may be used to show integration by parts. In this case, the first point is earned by columns (labeled or unlabeled) that begin with $t + 5$ and $\cos\left(\frac{t}{4}\right)$. The second point is earned for $4(t+5)\sin\left(\frac{t}{4}\right) - \int 4\sin\left(\frac{t}{4}\right) dt$ or a mathematically equivalent expression.
- The third point is earned only for an expression mathematically equivalent to $4(t+5)\sin\left(\frac{t}{4}\right) + 16\cos\left(\frac{t}{4}\right) + C$ (such as $4t\sin\left(\frac{t}{4}\right) + 20\sin\left(\frac{t}{4}\right) + 16\cos\left(\frac{t}{4}\right) + C$) in the presence of correct supporting work.
- To earn the third point a response must have a final answer that includes a constant of integration.
- Alternate solution:

$$\int (t+5)\cos\left(\frac{t}{4}\right) dt = \int t\cos\left(\frac{t}{4}\right) dt + \int 5\cos\left(\frac{t}{4}\right) dt$$

$$u = t \qquad dv = \cos\left(\frac{t}{4}\right) dt$$

$$du = dt \qquad v = 4\sin\left(\frac{t}{4}\right)$$

$$\int t\cos\left(\frac{t}{4}\right) dt + \int 5\cos\left(\frac{t}{4}\right) dt = 4t\sin\left(\frac{t}{4}\right) - \int 4\sin\left(\frac{t}{4}\right) dt + \int 5\cos\left(\frac{t}{4}\right) dt$$

$$= 4t\sin\left(\frac{t}{4}\right) + 16\cos\left(\frac{t}{4}\right) + 20\sin\left(\frac{t}{4}\right) + C$$

  - A response can earn the first and second points for correctly applying integration by parts to $\int t\cos\left(\frac{t}{4}\right) dt$. The tabular method may be used to show integration by parts. The third point is earned for the correct answer.

Total for part (d) 3 points

Total for question 5 9 points

Answer QUESTION 5 parts (a) and (b) on this page.

x	0	π	2π
$f'(x)$	5	6	0

Response for question 5(a)

$h(x) = \int_0^x \sqrt{1+(f'(t))^2}\,dt \quad h'(x) = \sqrt{1+(f'(x))^2}$

$h'(\pi) = \sqrt{1+(f'(\pi))^2} = \sqrt{1+(6)^2} = \sqrt{1+36} = \boxed{\sqrt{37}}$

Response for question 5(b)

The $\int_0^\pi \sqrt{1+(f'(x))^2}\,dx$ evaluates the arc length of $f(x)$ from $x=0$ to $x=\pi$.

5 5 5 5 5 NO CALCULATOR ALLOWED 5 5 5 5 5

Answer QUESTION 5 parts (c) and (d) on this page.

Response for question 5(c)

x	y	$f'(x)$	Δy
0	0	5	5π
π	5π	6	6π
2π	11π		

$f(2\pi) \approx 11\pi$

$\Delta y = \Delta x f'(x)$

$y = y_0 + \Delta y$

$(5)(\pi) = 5\pi \quad 5\pi + 0 = 5\pi$

$(6)(\pi) = 6\pi$

$f'(\pi) = 6 \quad f'(0) = 5$

$5\pi + 6\pi = 11\pi$

Response for question 5(d)

$\int (t+5)\cos\left(\frac{t}{4}\right)dt = (t+5)4\sin\left(\frac{t}{4}\right) - \int 4\sin\left(\frac{t}{4}\right)dt$

$= 4t\sin\left(\frac{t}{4}\right) + 20\sin\left(\frac{t}{4}\right) + 16\cos\left(\frac{t}{4}\right) + C$

$u = t+5 \quad dv = \cos\left(\frac{t}{4}\right)dt$

$du = dt \quad v = 4\sin\left(\frac{t}{4}\right)$

$\int (t+5)\cos\left(\frac{t}{4}\right)dt = 4t\sin\left(\frac{t}{4}\right) + 20\sin\left(\frac{t}{4}\right) + 16\cos\left(\frac{t}{4}\right) + C$

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Q5228/13

Answer QUESTION 5 parts (a) and (b) on this page.

x	0	π	2π
$f'(x)$	5	6	0

Response for question 5(a)

$h'(x) = \sqrt{1+(f'(x))^2}$ by fund thm of calc

$h'(\pi) = \sqrt{1+(f'(\pi))^2}$

$h'(\pi) = \boxed{\sqrt{37}}$

Response for question 5(b)

Accumulates on interval $0<x<\pi$

Answer QUESTION 5 parts (c) and (d) on this page.

Response for question 5(c)

	og y	slope * step	new
0	0	5π	5π
π	5π	6π	11π
2π	11π		

11π

Response for question 5(d)

$\int (t+5)\cos\left(\frac{t}{4}\right) dt$

$(t+5)\cdot 4\sin\left(\frac{t}{4}\right) + 16\cos\left(\frac{t}{4}\right)$

D	I
$t+5$ (+)	$\cos\left(\frac{t}{4}\right)$
1 (−)	$4\sin\left(\frac{t}{4}\right)$
0	$-16\cos\left(\frac{t}{4}\right)$

5 5 5 5 5 NO CALCULATOR ALLOWED 5 5 5 5 5

Answer QUESTION 5 parts (a) and (b) on this page.

x	0	π	2π
$f'(x)$	5	6	0

Response for question 5(a)

$h(x) = \int_0^x \sqrt{1+(f'(t))^2}\, dt$

$h'(x) = \sqrt{1+(f'(x))^2}$

$h'(\pi) = \sqrt{1+(f'(\pi))^2}$

$h'(\pi) = \sqrt{1+(6)^2} = \sqrt{37}$

Response for question 5(b)

$\int_0^\pi \sqrt{1+(f'(x))^2}$, provides the sum of the change in the function $\sqrt{1+(f'(x))^2}$ over the interval $0 \le x \le \pi$.

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

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Q5228/12

5 5 5 5 5 NO CALCULATOR ALLOWED 5 5 5 5 5

Answer QUESTION 5 parts (c) and (d) on this page.

Response for question 5(c)

$f(0)=0$

$f'(0)=5$

$y_1 = 0+5(x-0)$

$y_1 = 0+5(\pi-0)$

$y_1 = 5\pi$

$f(\pi)=5\pi$

$f'(\pi)=6$

$y_2 = 5\pi + 6(x-\pi)$

$y_2 = 5\pi + 6(\pi)$

$y_2 = 11\pi = f(2\pi)$

Response for question 5(d)

$\int (t+5)\cos\left(\frac{t}{4}\right)$

$u = t+5 \qquad v = \cos\frac{t}{4}$

$du = dt \qquad dv = -\frac{1}{4}\sin\frac{t}{4}\,dt$

$(t+5)(\cos t/4) - \int \frac{1}{4}\sin t/4\,dt$

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Q5228/13

Question 5

Note: Student samples are quoted verbatim and may contain spelling and grammatical errors.

Overview

In this question, students were told that the function f is twice differentiable and that $f(0) = 0.$ A table of selected values of x and $f'(x)$ was provided.

In part (a) students were told that for $x \geq 0,$ the function h is defined by $h(x) = \int_0^x \sqrt{1 + (f'(t))^2}\, dt$ and were asked to find the value of $h'(\pi).$ A correct response will use the Fundamental Theorem of Calculus to find $h'(x) = \sqrt{1 + (f'(x))^2},$ then will evaluate this expression at $x = \pi.$

In part (b) students were asked what information the expression $\int_0^\pi \sqrt{1 + (f'(x))^2}\, dx$ provides about the graph of $f.$ A correct response indicates that this is the expression for the arc length of the graph of f on the interval $[0, \pi].$

In part (c) students were told to use Euler's method to approximate $f(2\pi),$ starting at $x = 0,$ with two steps of equal size. A correct response will use the line tangent to the graph of f at $(0, 0)$ to find an approximation for $f(\pi),$ then use the line tangent to the graph of f at $(\pi, f(\pi))$ to approximate the value of $f(2\pi).$

In part (d) students were asked to find $\int (t + 5)\cos\left(\frac{t}{4}\right) dt.$ A correct response will use the technique of integration by parts to find an answer of $4(t + 5) \cdot \sin\left(\frac{t}{4}\right) + 16\cos\left(\frac{t}{4}\right) + C.$

Sample: 5A
Score: 9

The response earned 9 points: 2 points in part (a), 2 points in part (b), 2 points in part (c), and 3 points in part (d).

In part (a) the response earned the first point with the correct application of the Fundamental Theorem of Calculus in line 1 on the right. The response would have earned the second point with the expression $\sqrt{1 + (6)^2}.$ In this case, the response correctly simplifies to the expression $\sqrt{37}$ at the end of line 2 and earned the second point.

In part (b) the response earned the first point with the phrase "arc length of $f(x).$" The response earned the second point with the interval "from $x = 0$ to $x = \pi$" in line 2.

In part (c) the response earned the first point with two correct Euler's steps in the table. The response earned the second point with the boxed answer $f(2\pi) \approx 11\pi.$

In part (d) the response earned the first point with the correct identification of u and dv in line 2. The response earned the second point with the correct application of integration by parts on the right side of line 1. The response earned the third point with the boxed answer.

Question 5 (continued)

Sample: 5B
Score: 6

The response earned 6 points: 2 points in part (a), no points in part (b), 2 points in part (c), and 2 points in part (d).

In part (a) the response earned the first point with the correct application of the Fundamental Theorem of Calculus in line 1. The response earned the second point with the boxed solution $\sqrt{37}$ at the end of line 3.

In part (b) the response did not earn the first point with the phrase "Accumulates" because this phrase does not reference arc length. The response did not earn the second point because the second point cannot be earned without a connection to arc length.

In part (c) the response earned the first point with two correct Euler's steps in the table. The response earned the second point with the boxed answer of 11π.

In part (d) the response earned the first point with the table on the right which begins with the columns $t + 5$ and $\cos\left(\frac{t}{4}\right)$. The response earned the second point with the correct application of integration by parts on the left in line 2. The response did not earn the third point because the boxed answer does not include a constant of integration.

Sample: 5C
Score: 4

The response earned 4 points: 2 points in part (a), no points in part (b), 2 points in part (c), and no points in part (d).

In part (a) the response earned the first point with the correct application of the Fundamental Theorem of Calculus in line 2. The response would have earned the second point with the expression $\sqrt{1 + (6)^2}$ in line 4. In this case, the response correctly simplifies to the solution $\sqrt{37}$ at the end of line 4 and earned the second point.

In part (b) the response did not earn the first point with the phrase "the sum of the change in the function" because this phrase does not reference arc length. The response did not earn the second point because the second point cannot be earned without a connection to arc length.

In part (c) the response earned the first point with two correct Euler's steps in line 4 and line 9. The response earned the second point with the answer of $y_2 = 11\pi = f(2\pi)$ in line 10.

In part (d) the response did not earn the first point with the incorrect identification of dv in line 3. The response did not earn the second point because the response applies integration by parts with the incorrect function for v in line 4. The response did not earn the third point because the integration is not complete.

2024

AP

AP® Calculus BC

Sample Student Responses and Scoring Commentary

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Free-Response Question 6

☑ Scoring Guidelines

☑ Student Samples

☑ Scoring Commentary

Part B (BC): Graphing calculator not allowed
Question 6 — 9 points

General Scoring Notes

The model solution is presented using standard mathematical notation.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

The Maclaurin series for a function f is given by $\sum_{n=1}^{\infty} \frac{(n+1)x^n}{n^2 6^n}$ and converges to $f(x)$ for all x in the interval of convergence. It can be shown that the Maclaurin series for f has a radius of convergence of 6.

	Model Solution	Scoring	
(a)	Determine whether the Maclaurin series for f converges or diverges at $x = 6$. Give a reason for your answer.		
	At $x = 6$, the series is $\sum_{n=1}^{\infty} \frac{(n+1)6^n}{n^2 6^n} = \sum_{n=1}^{\infty} \frac{n+1}{n^2}$.	Considers $\frac{(n+1)6^n}{n^2 6^n}$	**1 point**
	Because $\frac{n+1}{n^2} > \frac{1}{n}$ for all $n \geq 1$ and the harmonic series $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges, the series $\sum_{n=1}^{\infty} \frac{n+1}{n^2}$ diverges by the comparison test.	Answer with reason	**1 point**

Scoring notes:

- To earn the first point using either the comparison or limit comparison test, a response must consider the term $\frac{(n+1)6^n}{n^2 6^n}$. This could be shown by considering the term $\frac{n+1}{n^2}$, either individually or as part of a sum.
- To earn the second point using the comparison test a response must demonstrate that the terms $\frac{n+1}{n^2}$ are larger than the terms in a divergent series.
  - o "$\frac{n+1}{n^2} > \frac{1}{n}$, diverges" earns both points.
  - o The response does not need to use the term "comparison test," but the response cannot declare use of an incorrect test.
- Alternate solution (limit comparison test):

  At $x = 6$, the series is $\sum_{n=1}^{\infty} \frac{(n+1)6^n}{n^2 6^n} = \sum_{n=1}^{\infty} \frac{n+1}{n^2}$.

  Because $\lim_{n\to\infty} \frac{\frac{n+1}{n^2}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n^2+n}{n^2} = 1$ and the harmonic series $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges, the series $\sum_{n=1}^{\infty} \frac{n+1}{n^2}$ diverges by the limit comparison test.

  - o To earn the second point using the limit comparison test, a response must correctly write the limit of the ratio of the terms in the given series to the terms of a divergent series and demonstrate that the limit of this ratio is 1.
  - o The response does not need to use the term "limit comparison test," but the response cannot declare use of an incorrect test.

Total for part (a) 2 points

(b) It can be shown that $f(-3) = \sum_{n=1}^{\infty} \frac{(n+1)(-3)^n}{n^2 6^n} = \sum_{n=1}^{\infty} \frac{n+1}{n^2}\left(-\frac{1}{2}\right)^n$ and that the first three terms of this series sum to $S_3 = -\frac{125}{144}$. Show that $|f(-3) - S_3| < \frac{1}{50}$.

Solution	Scoring
$f(-3) = \sum_{n=1}^{\infty} \frac{(n+1)(-3)^n}{n^2 6^n} = \sum_{n=1}^{\infty} \frac{(n+1)}{n^2}\left(-\frac{1}{2}\right)^n$ is an alternating series with terms that decrease in magnitude to 0.	Uses fourth term **1 point**
By the alternating series error bound, $\sum_{n=1}^{3} \frac{n+1}{n^2}\left(-\frac{1}{2}\right)^n = -\frac{125}{144}$ approximates $f(-3)$ with error of at most $\left\lvert \frac{4+1}{4^2}\left(-\frac{1}{2}\right)^4 \right\rvert = \frac{5}{256} < \frac{5}{250} = \frac{1}{50}$. Thus, $\lvert f(-3) - S_3 \rvert < \frac{1}{50}$.	Verification **1 point**

Scoring notes:

- The first point is earned for correctly using $x = -3$ in the fourth term. (Listing the fourth term as part of a polynomial is not sufficient.) Using $x = -3$ in any term of degree five or higher does not earn this point.
- The expression $\frac{4+1}{4^2}\left(-\frac{1}{2}\right)^4$ earns the first point, but just $\frac{5}{256}$ does not earn the first point.
- A response including the expression $\frac{4+1}{4^2}\left(-\frac{1}{2}\right)^4$ that is subsequently simplified incorrectly earns the first point but not the second.
- To earn the second point the response must state that the series for $f(-3)$ is alternating or that the alternating series error bound is being used.
    - A response of just "Error $\le \frac{4+1}{4^2}\left(-\frac{1}{2}\right)^4 < \frac{1}{50}$" (or any equivalent mathematical expression) earns both points, provided it is accompanied by an indication that the series is alternating.
- A response that declares the error is equal to $\frac{5}{256}$ (or any equivalent form of this value) does not earn the second point.

Total for part (b) 2 points

(c) Find the general term of the Maclaurin series for f', the derivative of f. Find the radius of convergence of the Maclaurin series for f'.

The general term of the Maclaurin series for f' is $\dfrac{(n+1)nx^{n-1}}{n^2 6^n} = \dfrac{(n+1)x^{n-1}}{n \cdot 6^n}$.	General term	**1 point**
Because the radius of convergence of the Maclaurin series for f is 6, the radius of convergence of the Maclaurin series for f' is also 6.	Radius	**1 point**

Scoring notes:

- A response of $\dfrac{(n+1)nx^{n-1}}{n^2 6^n}$ earns the first point. Any expression mathematically equivalent to this also earns the first point.
- The response need not simplify $\dfrac{(n+1)nx^{n-1}}{n^2 6^n}$, but any presented simplification must be correct in order to earn the first point.
- The second point is earned only for a supported answer of 6. The second point can be earned without the first.
- Alternate solution for second point (ratio test):

$$\lim_{n\to\infty}\left|\frac{\dfrac{(n+2)x^n}{(n+1)6^{n+1}}}{\dfrac{(n+1)x^{n-1}}{n\cdot 6^n}}\right| = \lim_{n\to\infty}\left|\frac{n(n+2)}{(n+1)^2}\cdot\frac{x}{6}\right| = \frac{|x|}{6} < 1 \Rightarrow |x| < 6$$

Therefore, the radius of convergence of the Maclaurin series for f' is 6.

- Alternate solution for second point (root test):

$$\lim_{n\to\infty}\sqrt[n]{\frac{(n+1)|x|^{n-1}}{n\cdot 6^n}} = \lim_{n\to\infty}\left(\frac{n+1}{n}\right)^{1/n}\cdot|x|^{-1/n}\cdot\frac{|x|}{6} = 1\cdot 1\cdot\frac{|x|}{6} < 1 \Rightarrow |x| < 6$$

Therefore, the radius of convergence of the Maclaurin series for f' is 6.

Total for part (c) 2 points

(d) Let $g(x) = \sum_{n=1}^{\infty} \frac{(n+1)x^{2n}}{n^2 3^n}$. Use the ratio test to determine the radius of convergence of the Maclaurin series for g.

$\left\| \dfrac{\dfrac{(n+2)x^{2n+2}}{(n+1)^2 3^{n+1}}}{\dfrac{(n+1)x^{2n}}{n^2 3^n}} \right\| = \left\| \dfrac{(n+2)x^{2n+2}}{(n+1)^2 3^{n+1}} \cdot \dfrac{n^2 3^n}{(n+1)x^{2n}} \right\|$	Sets up ratio	**1 point**
$\lim_{n\to\infty} \left\| \dfrac{(n+2)n^2}{(n+1)^3} \right\| \cdot \left\| \dfrac{x^2}{3} \right\| = \left\| \dfrac{x^2}{3} \right\|$	Limit	**1 point**
$\left\| \dfrac{x^2}{3} \right\| < 1 \Rightarrow x^2 < 3 \Rightarrow \|x\| < \sqrt{3}$ The radius of convergence of g is $\sqrt{3}$.	Radius of convergence	**1 point**

Scoring notes:

- The first point is earned by presenting a correct ratio with or without absolute values. Once earned, this point cannot be lost. Any errors in simplification or evaluation of the limit will not earn the second point.
- The first point is earned for ratios mathematically equivalent to any of the following:

$$\frac{\frac{(n+2)x^{2n+2}}{(n+1)^2 3^{n+1}}}{\frac{(n+1)x^{2n}}{n^2 3^n}}, \quad \frac{(n+2)x^{2n+2}}{(n+1)^2 3^{n+1}} \cdot \frac{n^2 3^n}{(n+1)x^{2n}}, \quad \frac{\frac{(n+1)x^{2n}}{n^2 3^n}}{\frac{nx^{2n-2}}{(n-1)^2 3^{n-1}}}, \text{ or } \frac{(n+1)x^{2n}}{n^2 3^n} \cdot \frac{(n-1)^2 3^{n-1}}{nx^{2n-2}}.$$

- The first point is also earned for ratios mathematically equivalent to the following reciprocal ratios:

$$\frac{\frac{(n+1)x^{2n}}{n^2 3^n}}{\frac{(n+2)x^{2n+2}}{(n+1)^2 3^{n+1}}}, \quad \frac{(n+1)x^{2n}}{n^2 3^n} \cdot \frac{(n+1)^2 3^{n+1}}{(n+2)x^{2n+2}}, \quad \frac{\frac{nx^{2n-2}}{(n-1)^2 3^{n-1}}}{\frac{(n+1)x^{2n}}{n^2 3^n}}, \text{ or } \frac{nx^{2n-2}}{(n-1)^2 3^{n-1}} \cdot \frac{n^2 3^n}{(n+1)x^{2n}}.$$

  Responses including any of these reciprocal ratios can earn the second point for using limit notation to correctly find a limit of the absolute value of their ratio to be $\left|\frac{3}{x^2}\right|$. Such responses earn the third point only for a final answer of $\sqrt{3}$ with a valid explanation for reporting the reciprocal of $\frac{1}{\sqrt{3}}$.
- To earn the second point a response must use the ratio and correctly evaluate the limit of the ratio, using correct limit notation.
- The third point is earned only for an answer of $\sqrt{3}$ with supporting work.

Total for part (d) 3 points

Total for question 6 9 points

6 6 6 6 6 NO CALCULATOR ALLOWED 6 6 6 6 6

Answer QUESTION 6 parts (a) and (b) on this page.

Response for question 6(a)

$x=6: \sum_{n=1}^{\infty} \frac{(n+1)6^n}{n^2 6^n} = \sum_{n=1}^{\infty} \frac{(n+1)}{n^2}$

compare with $\sum_{n=1}^{\infty} \frac{1}{n}$ Harmonic series diverges

$\lim_{n\to\infty} \left| \frac{n+1}{n^2} \cdot \frac{n}{1} \right| = 1 > 0$

$\sum_{n=1}^{\infty} \frac{(n+1)6^n}{n^2 6^n}$ diverges by the LCT.

Response for question 6(b)

f(-3) is an alternating series whose terms decrease in absolute value to 0.

$|\text{Error}| \le \left| \frac{5}{16}\left(-\frac{1}{2}\right)^4 \right| < \frac{1}{50}$

Answer QUESTION 6 parts (c) and (d) on this page.

Response for question 6(c)

general term for $f' = \dfrac{n(n+1)x^{n-1}}{n^2 6^n}$

$$\lim_{n\to\infty}\left|\frac{(n+2)x^n}{(n+1)6^{n+1}}\cdot\frac{n6^n}{(n+1)x^{n-1}}\right| = \left|\frac{x}{6}\right| < 1$$

$-6 < x < 6$

$R = 6$

Response for question 6(d)

$$\lim_{n\to\infty}\left|\frac{(n+2)x^{2n+2}}{(n+1)^2 3^{n+1}}\cdot\frac{n^2 3^n}{(n+1)x^{2n}}\right| = \left|\frac{x^2}{3}\right| < 1$$

$-3 < x^2 < 3$

$|x| < \sqrt{3}$

$-\sqrt{3} < x < \sqrt{3}$

$R = \sqrt{3}$

6 6 6 6 6 NO CALCULATOR ALLOWED 6 6 6 6 6

Answer QUESTION 6 parts (a) and (b) on this page.

Response for question 6(a)

at $x=6$

$\sum_{n=1}^{\infty} \frac{(n+1)6^n}{n^2 6^n} = \sum_{n=1}^{\infty} \frac{(n+1)}{n^2}$

$a_n = \frac{1}{n}$ diverges

By LCT:

$a_n = \frac{1}{n}$

$b_n = \frac{n+1}{n^2}$

$\lim_{n\to\infty} \frac{n+1}{n^2} \cdot \frac{n}{1} = \lim_{n\to\infty} \frac{n+1}{n} = 1$ so LCT applies because 1 is finite and positive

at $x=6$ the Maclaurin series diverges when using the Limit comparison test

Response for question 6(b)

$a_n = \frac{n+1}{n^2}\left(-\frac{1}{2}\right)^n$

$a_4 = \frac{4+1}{4^2}\left(\frac{-1}{2}\right)^4$

$= \left(\frac{5}{16}\right)\left(\frac{1}{16}\right) = \frac{5}{16^2}$

$a_4 = \frac{5}{256}$

$|S - S_n| < a_{n+1}$

$|f(-3) - S_3| < a_4$

because $a_4 = \frac{5}{256}$ and $|f(-3) - S_3| < a_4$ and $\frac{5}{256} < \frac{1}{50}$ $|f(-3) - S_3| < \frac{1}{50}$

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

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Q5228/14

6 6 6 6 6 **NO CALCULATOR ALLOWED** 6 6 6 6 6

Answer QUESTION 6 parts (c) and (d) on this page.

Response for question 6(c)

anything w/ n is a constant

f general term $= \dfrac{(n+1)(x^n)}{n^2 6^n}$

f' general term $= \dfrac{(n+1)}{n^2 6^n} \cdot nx^{n-1} = \dfrac{(n+1)(nx^{n-1})}{n^2 6^n}$

ratio test:

$\lim_{n\to\infty} \left| \dfrac{((n+1)+1)((n+1)x^{n+1-1})}{(n+1)^2 6^{n+1}} \cdot \dfrac{n^2 6^n}{(n+1)(nx^{n-1})} \right| < 1 =$

$\lim_{n\to\infty} \left| \dfrac{(n+2)(x^n)\, n^2}{(n+1)^2\, 6\, (n+1)(n x^{n-1})} \right| < 1 \Rightarrow \lim_{n\to\infty} \left|\dfrac{x}{6}\right| \cdot \lim_{n\to\infty} \left| \dfrac{(n+2)(n^2)}{(n+1)^2 (n)} \right|$

$\lim_{n\to\infty} \left|\dfrac{x}{6}\right| < 1 \Rightarrow \left|\dfrac{x}{6}\right| < 1 \Rightarrow |x| < 6$

$R = 6$ the radius of convergence is $\boxed{6}$ for f'

Response for question 6(d)

$\lim_{n\to\infty} \left| \dfrac{a_{n+1}}{a_n} \right| < 1$

ratio test:

$\lim_{n\to\infty} \left| \dfrac{((n+1)+1)(x^{2(n+1)})}{(n+1)^2\, 3^{n+1}} \cdot \dfrac{n^2 3^n}{(n+1)(x^{2n})} \right| < 1 =$

$\lim_{n\to\infty} \left| \dfrac{(n+2)(x^{2n+2})\, n^2 3^n}{(n+1)^3\, (x^{2n})} \right| < 1 = \lim_{n\to\infty} \left| \dfrac{x^2}{3} \dfrac{(n+2)(n^2)}{(n+1)^3} \right| < 1$

$= \lim_{n\to\infty} \left|\dfrac{x^2}{3}\right| \cdot \lim_{n\to\infty} \left| \dfrac{(n+2)(n^2)}{(n+1)^3} \right| < 1 \Rightarrow \left|\dfrac{x^2}{3}\right| < 1$ (becomes 1)

The radius of convergence is $\boxed{\sqrt{3}}$

$= |x^2| < 3$

$|x| < \sqrt{3}$

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Answer QUESTION 6 parts (a) and (b) on this page.

Response for question 6(a)

$x=6$, $\sum_{n=1}^{\infty} \frac{(n+1)6^n}{n^2 6^n} \to \frac{(n+1)}{n^2} \to$ behaves like $\sum_{n=1}^{\infty} \frac{1}{n} = b_n$, which diverges. Since $b_n < \sum_{n=1}^{\infty} \frac{n+1}{n^2}$, then the series must also diverge.

Response for question 6(b)

$f(-3) = \sum_{n=1}^{\infty} \frac{n+1}{n^2} \left(-\frac{1}{2}\right)^n$ $\quad \frac{a}{1-r} = \frac{\frac{n+1}{n^2}}{1-(-\frac{1}{2})} = \frac{\frac{n+1}{n^2}}{\frac{3}{2}} = \frac{2(n+1)}{3n^2}$

$\frac{2(3+1)}{3(3)^2} = \frac{8}{27}$ $\quad f(-3) = -1 + \frac{3}{8} - \frac{4}{72} + \frac{5}{16 \cdot 32}$

$\left| \frac{8}{27} - \left(-\frac{125}{144}\right) \right| =$

6 6 6 6 6 NO CALCULATOR ALLOWED 6 6 6 6 6

Answer QUESTION 6 parts (c) and (d) on this page.

Response for question 6(c)

$f(x) = \frac{(n+1)x^n}{n^2 6^n} = \frac{x^n n + x^n}{n^2 6^n} \quad f'(x) = \frac{x^n n + x^n}{n^2 6^n}$

Response for question 6(d)

$g(x) = \sum_{n=1}^{\infty} \frac{(n+1)x^{2n}}{n^2 3^n} \rightarrow \lim_{n\to\infty} \left| \frac{(n+2)x^{2n+2}}{(n+1)^2 3^{n+1}} \cdot \frac{n^2 \cdot 3^n}{(n+1)x^{2n}} \right| = \lim_{n\to\infty} \left| \frac{(n+2)x^2 n^2}{(n+1)^3 \cdot 3} \right|$

$= \lim_{n\to\infty} \left| \frac{x^2 n^3 + 2x^2 n^2}{3(n+1)^3} \right| = \left| \frac{x^2}{3} \right| < 1 \rightarrow -1 < \frac{x^2}{3} < 1 \rightarrow -3 < x^2 < 3$

$\rightarrow \boxed{\sqrt{-3} < x < \sqrt{3}}$

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Q5229/15

Question 6

Note: Student samples are quoted verbatim and may contain spelling and grammatical errors.

Overview

In this question, students are told that the Maclaurin series for a function f is given by $\sum_{n=1}^{\infty} \frac{(n+1)x^n}{n^2 6^n}$. Furthermore, the series converges to $f(x)$ for all x in the interval of convergence and the radius of that interval of convergence is 6.

In part (a) the students are asked to determine whether the Maclaurin series for f converges or diverges at $x = 6$ and to give a reason for their answer. A correct response will determine that at $x = 6,$ the series is $\sum_{n=1}^{\infty} \frac{(n+1)}{n^2}$ and that for all $n \geq 1,$ the terms of this series are larger than the terms of the divergent harmonic series. Therefore, this series diverges by the comparison test.

In part (b) students are told that $f(-3) = \sum_{n=1}^{\infty} \frac{(n+1)}{n^2} \cdot \left(-\frac{1}{2}\right)^n$ and that the first three terms of this series sum to $S_3 = -\frac{125}{144}.$ Students are asked to show that $\left|f(-3) - S_3\right| < \frac{1}{50}.$ A correct response will observe that the series for $f(-3)$ is alternating with terms that decrease in magnitude to $0.$ Therefore, by the alternating series error bound, S_3 approximates $f(-3)$ with an error that is no more than the value of the fourth term of the series. The fourth term is $\left|\frac{4+1}{4^2} \cdot \left(-\frac{1}{2}\right)^4\right| = \frac{5}{256},$ which is less than $\frac{1}{50}.$

In part (c) students are asked to find the general term of the Maclaurin series for f' and to find the radius of convergence of the Maclaurin series for $f'.$ A correct response will differentiate the general term of the Maclaurin series for f to find a general term of $\frac{(n+1)x^{n-1}}{n \cdot 6^n}$ and will note that the radius of convergence of the Maclaurin series for f' must be $6,$ because this is the radius of convergence of the Maclaurin series of $f.$

In part (d) students are given a new Maclaurin series, $g(x) = \sum_{n=1}^{\infty} \frac{(n+1)x^{2n}}{n^2 3^n},$ and asked to use the ratio test to determine the radius of convergence. A correct response will set up a ratio of consecutive terms, $\frac{a_{n+1}}{a_n},$ find the limit of the absolute value of that ratio as $n \to \infty,$ and determine for what values of x the limit is less than $1.$

Question 6 (continued)

Sample: 6A
Score: 9

The response earned 9 points: 2 points in part (a), 2 points in part (b), 2 points in part (c), and 3 points in part (d).

In part (a) the response earned the first point in the first line for the term $\dfrac{(n+1)6^n}{n^2\,6^n}$. The response earned the second point in the second, third, and fourth lines by correctly evaluating the limit of the ratio of our series with the harmonic series, stating that the "harmonic series diverges", and concluding that our series diverges.

In part (b) the response earned the first point in the second line for the term $\left|\dfrac{5}{16}\left(-\dfrac{1}{2}\right)^4\right|$. The response earned the second point by stating "$f(-3)$ is an alternating series" and presenting the inequality $|\text{Error}| \le \left|\dfrac{5}{16}\left(-\dfrac{1}{2}\right)^4\right| < \dfrac{1}{50}$.

In part (c) the response earned the first point in the first line for the correct general term of the derivative $\dfrac{n(n+1)x^{n-1}}{n^2\,6^n}$. The response earned the second point in the second and fourth lines for the limit of the ratio with absolute values, correct evaluation, and the answer.

In part (d) the response earned the first point in the first line for the term $\left|\dfrac{(n+2)x^{2n+2}}{(n+1)^2\,3^{n+1}} \cdot \dfrac{n^2\,3^n}{(n+1)x^{2n}}\right|$. The response earned the second point in the first line by correctly evaluating the limit with the term $\left|\dfrac{x^2}{3}\right|$. The response earned the third point for the correct answer of $\sqrt{3}$ with correct supporting work.

Sample: 6B
Score: 6

The response earned 6 points: 1 point in part (a), 1 point in part (b), 2 points in part (c), and 2 points in part (d).

In part (a) the response earned the first point in the second line on the left for the term $\dfrac{(n+1)6^n}{n^2\,6^n}$. The response did not earn the second point because the response did not reference that the series $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n}$ diverges and instead only referenced the term $\dfrac{1}{n}$.

In part (b) the response earned the first point in the second line on the right for the term $\dfrac{4+1}{4^2}\left(\dfrac{-1}{2}\right)^4$. The response did not earn the second point since the response did not reference alternating series or alternating series error bound.

Question 6 (continued)

In part (c) the response earned the first point in the second line for the term $\frac{(n+1)}{n^2 6^n} \cdot nx^{n-1}$. The response earned the second point for a correct application of the ratio test together with the correct answer.

In part (d) the response earned the first point in the third line for the term $\left| \frac{((n+1)+1)\left(x^{2(n+1)}\right)}{(n+1)^2 3^{n+1}} \cdot \frac{n^2 3^n}{(n+1)\left(x^{2n}\right)} \right|$.

The response did not earn the second point because in the fourth line there is a missing 3^{n+1} in the denominator. The response earned the third point for a correct radius of convergence with correct supporting work.

Sample: 6C
Score: 3

The response earned 3 points: 1 point in part (a), no points in part (b), no points in part (c), and 2 points in part (d).

In part (a) the response earned the first point in the first line for the term $\frac{(n+1)6^n}{n^2 6^n}$. The response did not earn the second point because the response presented an inequality that compares series instead of terms of series.

In part (b) the response did not earn the first point because the response did not correctly use the fourth term of the series. The response did not earn the second point because the response did not reference alternating series.

In part (c) the response did not earn the first point because the presented derivative is incorrect. The response did not earn the second point because the response did not present a radius of convergence.

In part (d) the response earned the first point in the first line for the term $\left| \frac{(n+2)x^{2n+2}}{(n+1)^2 3^{n+1}} \cdot \frac{n^2 \cdot 3^n}{(n+1)x^{2n}} \right|$. The response earned the second point in the second line for the correct evaluation of the limit as $\left| \frac{x^2}{3} \right|$. The response did not earn the third point because the response presented an imaginary number $\sqrt{-3}$ in the third line.